Verifying Trigonometric Identities

Section Objectives: Students will know how to verify trigonometric identities.

Strategy for Verifying Identities

1. Work with only one side of the equation at a time. Usually it is better to start with the more complicated side first.
2. Look for opportunities to factor an expression, add fractions, square a two-term quantity, or create a single-term denominator.
3. Look for opportunities to use the fundamental identities. Note which functions are in the final expression you want. Sine and cosine pair well, as do secant and tangent, and cosecant and cotangent.
4. As a last resort, convert all terms to sine and cosine.
5. Always try something. Even paths that lead to dead ends give you insight.

Ex: Verify the identity $\frac{\sec^2\theta - 1}{\sec^2\theta} = \sin^2\theta$

Ex: Verify the identity $2\sec^2\beta = \frac{1}{1-\sin\beta} + \frac{1}{1+\sin\beta}$

Ex: Verify the identity $(\tan^2 x + 1)(\cos^2 x - 1) = -\tan^2 x$

Ex: Verify the identity $\tan x + \cot x = \sec x \csc x$

Ex: Verify the identity $\sec x + \tan x = \frac{\cos x}{1 - \sin x}$

Ex: Verify the identity $\frac{\cot^2\theta}{1+\csc\theta} = \frac{1-\sin\theta}{\sin\theta}$

Some Important Examples from Calculus

Ex: Verify each identity

a. $\tan^4 x = \tan^2 x \sec^2 x - \tan^2 x$

b. $\sin^3 x \cos^4 x = (\cos^4 x - \cos^6 x)\sin x$

c. $\csc^4 x \cot x = \csc^2 x(\cot x + \cot^3 x)$