Transformations and Sketching Graphs
Section Objectives: Students will know how to identify and graph shifts, reflections, and nonrigid transformations of functions.

Translations
Many functions have graphs that are simple transformations of common graphs found in the last section.
Look at the graphs of the following to see how the graphs change:

$$f(x) = x^2 \text{ , } g(x) = (x-2)^2 \text{ , } h(x) = x^2 + 2$$

Let *c and d* be a positive real numbers. The following changes in the function $y = f(x)$ will produce the stated shifts in the graph of $y = f(x)$.

1. $h(x) = f(x - c)$ **Horizontal shift** *c* units to the **right**
2. $h(x) = f(x + c)$ **Horizontal shift** *c* units to the **left**
3. $h(x) = f(x) - d$ **Vertical shift** d units **downward**
4. $h(x) = f(x) + d$ **Vertical shift** d units **upward**

Ex: Given $f(x) = x^3$, describe the shifts of the graph of *f* generated by the following functions.
a) $g(x) = (x + 1)^3 + 1$. **b)** $h(x) = (x - 2)^3 - 4$.

Reflecting Graphs
The second common type of transformation is a reflection
Consider the graph of $\mathbf{f(x) = -x^2}$
The following changes in the function $y = f(x)$ will produce the stated reflections in the graph of $y = f(x)$.
1. $h(x) = -f(x)$: **reflection** in the ***x*-axis**
2. $h(x) = f(-x)$: **reflection** in the ***y*-axis**
Ex: Given $f(x) = x^3$, describe the reflections of the graph of *f* generated by the following functions.
a) $g(x) = (-x)^3 + 3$. **b)** $h(x) = -x^3 + 3$
Ex: Write the equation of the graph of $f(x) = x^4$ that is reflected along the x – axis and moved two units to upward.
Ex: Graph the following $\sqrt{x}$, $\sqrt{x-2}$, $\sqrt{x}+2$, $-\sqrt{x}$

Nonrigid Transformations

Horizontal and vertical shifts, and reflections are **Rigid Transformations**, they change position but shape is preserved. **Nonrigid transformations** actually *distort* the shape of a graph, instead of just shifting or reflecting it.

A nonrigid transformation of $y = f(x)$ comes from equations of the form $g(x) = k\,f(x)$.

If $|k| > 1$, there is a **vertical stretch** of $y = f(x)$.

If $0 < |k| < 1$, there is a **vertical shrink** of $y = f(x)$.

Ex: Compare the graph of each function with the graph of $f(x) = x^2$

(a) $g(x) = 4x^2$ **(b)** $h(x) = \frac{1}{4}x^2$