The Sine and Cosine Functions

Section Objectives: Students will know how to evaluate trigonometric functions.

The unit circle is a circle centered at the origin with radius 1 so the equation of this circle would be $x^2 + y^2 = 1$. We use this circle to help us define the six trigonometric functions.

We start by working with the two most basic trigonometric functions the **sine of θ**, written as**, sin(θ)** or **sinθ** and **cosine of θ** written as **cos(θ)** or **cosθ**, where θ is an angle in standard position.

> If **θ** is an angle in standard position and (x,y) is the point of intersection of the terminal side and the unit circle, then
>
> **sinθ = y** and **cosθ = x**

The domain of both sine and cosine is the set of all angles in standard position or $(-\infty,\infty)$ and the range for each is $[-1,1]$

Multiples of 90^0 or $\pi/2$

To start evaluating these functions around the unit circle we work with the easiest points, the ones that fall on the axis. Points (0,1),(1,0),(-1,0), and (0,-1) are points on the unit circle. This means we can use the previous definition to evaluate the angles that have their terminal side pass through each one of these points.

Ex: Find the exact values of:

a. $\sin(0)$ **b.** $\cos(\frac{\pi}{2})$ **c.** $\sin(-270^0)$

Multiples of 45^0 or $\pi/4$

To find the points associated with the 45^0 angle we look at the line y = x since the terminal side of the 45^0 angle falls on this line. Since y = x we can rewrite $x^2 + y^2 = 1$ as $x^2 + x^2 = 1$ and solve for x, $x = \pm\sqrt{2}/2$ so y equals the same.

Ex: Find the exact values of:

a. $\sin(\frac{\pi}{4})$ **b.** $\cos(\frac{-\pi}{4})$ **c.** $\sin(135^0)$

Multiples of 30^0 or $\pi/6$

Since 30^0 and 60^0 are related we can find the points for each of these at the same time using a little geometry. We can create an equilateral triangle by connecting the point on the unit circle (x,y) the origin and point (1,0) and this will give us our 60^0 angle. By splitting one side of the triangle in half we see that the x value on the terminal side of the 60^0 angle must be ½, we can then use this in the unit circle equation $x^2 + y^2 = 1$ to solve for y giving us $y = \pm\sqrt{3}/2$.

Ex: Find the exact values of:

a. $\sin(\frac{\pi}{6})$ **b.** $\cos(\frac{\pi}{3})$ **c.** $\sin(-150^0)$

Knowing which quadrant your angle falls and whether the x and y values are positive or negative is key in knowing whether the sign of the sine and cosine functions.

Reference Angles

If θ is a nonquadrantal angle in standard position, then the **reference angle** for θ is the positive angle θ', "theta prime", formed by the terminal side of θ and the positive or negative x-axis.

In other words, if θ is in standard position, then the **reference angle** is the acute angle formed by the terminal side of θ and the *x*-axis.

Tip: *The reference angle uses only the x-axis.*

Ex: Find the reference angle θ' for the following angles.
a. θ = 125^0 **b.** θ = 2.3 **c.** θ = 2π/3

For an angle θ in standard position that is not a quadrantal angle, the value of a trigonometric function of θ can be found by finding the value for its reference angle and affixing the appropriate sign.

The Fundamental Identity or Pythagorean Identity

An identity is an equation that is satisfied for all values of the variable for which both sides are defined.

The Fundamental Identity

If θ is any angle of real number then,

$$\sin^2\theta + \cos^2\theta = 1$$

This comes from $x^2 + y^2 = 1$ since sinθ = y and cosθ = x.

Ex: Find cosθ given that sinθ = 3/5 and θ is an angle in quad II