

The Four Other Trigonometric Functions and Their Graphs

Section Objectives: Students will know how to evaluate and graph the four other trigonometric functions.

There are four other trigonometric functions. They are tangent (tan), cotangent (cot), secant (sec), and cosecant (csc). These can also be defined using the unit circle.

If θ is an angle in standard position and (x,y) is the point of intersection of the terminal side and the unit circle, we define the tangent, cotangent, secant, and cosecant as

$$\tan \theta = \frac{y}{x}, \quad \cot \theta = \frac{x}{y}, \quad \sec \theta = \frac{1}{x}, \quad \csc \theta = \frac{1}{y}$$

Since these functions are equal to fractional values we need to restrict the domains to keep the denominators from being zero.

Since $\sin \theta = y$ and $\cos \theta = x$ we can rewrite each of the previous in terms of sine and cosine

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \cot \theta = \frac{\cos \theta}{\sin \theta}, \quad \sec \theta = \frac{1}{\cos \theta}, \quad \csc \theta = \frac{1}{\sin \theta}$$

again providing that the denominator is not zero.

$\cos \theta = 0$ when the x value is zero. This happens when the terminal side of θ goes through points $(0,1)$ and $(0,-1)$ which corresponds to

$\theta = \frac{\pi}{2}$, $\theta = \frac{3\pi}{2}$ and anything coterminal to those angles. Therefore the

domain for tangent and secant is $\left\{ \theta \mid \theta \neq \frac{\pi}{2} + k\pi, \text{ where } k \text{ is an integer} \right\}$

$\sin \theta = 0$ when the y value is zero. This happens when the terminal side of θ goes through $(1,0)$ and $(-1,0)$ which corresponds to $\theta = 0$ and $\theta = \pi$

and anything coterminal to those angles. Therefore the domain for cotangent and cosecant is $\left\{ \theta \mid \theta \neq k\pi, \text{ where } k \text{ is an integer} \right\}$

Signs of the trigonometric functions in the four quadrants.

- Both x and y are positive in the first quadrant, all six functions are positive in the first quadrant.
- Only y is positive in the second quadrant, only sine and cosecant are positive in the second quadrant.
- Both x and y are negative in the third quadrant, only tangent and cotangent are positive in the third quadrant.
- Only x is positive in the fourth quadrant, only cosine and secant are positive in the fourth quadrant.

- **Ex:** Find the value for all six trigonometric functions for each angle.

a. $\frac{\pi}{3}$ b. 135° c. $-\frac{\pi}{6}$ d. $-\frac{\pi}{2}$

Graphs if the other Trig functions

The **tangent function** is periodic with **period π** .

Functions that are fractions can have vertical asymptotes at points at which the denominator is zero and the numerator is not.

The line $x = a$ is a **vertical asymptote** of the graph of **$f(x)$** if $f(x) \rightarrow \infty$ or $f(x) \rightarrow -\infty$ as $x \rightarrow a$, either from the right or from the left.

Therefore, because **$\tan x = \sin x / \cos x$** , the graph of $y = \tan x$ will have **vertical asymptotes at** $\left\{ \theta \mid \theta = \frac{\pi}{2} + k\pi, \text{ where } k \text{ is an integer} \right\}$.

Graph of $y = \tan x$ has:

Period: π

Domain: $\left\{ \theta \mid \theta \neq \frac{\pi}{2} + k\pi, \text{ where } k \text{ is an integer} \right\}$

Range: $(-\infty, \infty)$

Vertical Asymptote: $\left\{ \theta \mid \theta = \frac{\pi}{2} + k\pi, \text{ where } k \text{ is an integer} \right\}$

Key points of the graph of the tangent function are the asymptotes at the ends of the period and the intercept in the middle.

Sketching $y = d + a \tan (b(x - c))$ is very similar to the process for $\sin x$

2 consecutive asymptotes can be found by solving

$$b(x - c) = -\frac{\pi}{2} \quad \text{and} \quad b(x - c) = \frac{\pi}{2}$$

Which means the period is the distance between the asymptotes.

And the **midpoint** between the asymptotes is the **$x - \text{int}$** .

Ex: Sketch the graph of

$$y = \tan \frac{\pi}{2} \quad \text{and} \quad y = -3 \tan 2x$$

Graph of the Cotangent Function

Similar to the tangent function, the **cotangent function** periodic with **period π** , and has **asymptotes at $x = \pi n$** .

Because $y = \cot x = \cos x / \sin x$ and $\sin x = 0$ when $x = n\pi$, $y = \cot x$ will have **vertical asymptotes at $\{\theta | \theta = k\pi, \text{ where } k \text{ is an integer}\}$**

The Graph of $y = \cot x$ has:

Period: π

Domain: $\{\theta | \theta \neq k\pi, \text{ where } k \text{ is an integer}\}$

Range: $(-\infty, \infty)$

Vertical Asymptotes: $\{\theta | \theta = k\pi, \text{ where } k \text{ is an integer}\}$

Key points for the $\cot x$ are the vertical asymptotes and the x-ints for the graph of $y = d + a \cot(b(x - c))$

2 consecutive asymptotes can be found by solving

$$b(x - c) = 0 \quad \text{and} \quad b(x - c) = \pi$$

Which means the period is the distance between the asymptotes.

And the **midpoint** between the asymptotes is the **x - int**.

Ex: Sketch the graph of $y = 2 \cot \frac{x}{3}$

Graphs of the Reciprocal Functions

Remember:

$$\csc x = \frac{1}{\sin x} \quad \text{and} \quad \sec x = \frac{1}{\cos x}$$

This implies that if **$\sin x = 0$** then **$\csc x$** has a vert. asym. and if **$\cos x = 0$** then **$\sec x$** has a vert. asym.

Also where sine has a relative maximum, the cosecant has a relative minimum, and where the sine has a relative minimum, the cosecant has a relative maximum. Similarly for the cosine and secant. The easiest way to draw these graphs is to draw the sine or cosine graphs and fill in the secant or cosecant graph from there.

Ex: Sketch the graphs of $y = 2 \csc\left(x + \frac{\pi}{4}\right)$ and $y = \sec 2x$