Calc I Test #3 Problem Set G. Buthusiem. KV

1. Evaluate $\int_0^1 3x\,dx$ using $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

2. Evaluate the following indefinite integrals:

a. $\int \frac{x^2+2x-6}{x^4}\,dx$ b. $\int (5\cos x - 2\sec^2 x)\,dx$

c. $\int 6x^3\sqrt{3x^4+2}\,dx$ d. $\int \frac{x+4}{x^2+8x-7}\,dx$ e. $\int \sin^4 5x\cos 5x\,dx$

3. Evaluate the following definite integrals:

a. $\int_0^3 (5x^3+3x-1)\,dx$ b. $\int_0^1 (3x+1)^5\,dx$ c. $\int_{-\frac{\pi}{4}}^{\pi/4} \sin 2x\,dx$

4. Find the area bounded by $f(x) = -x^2+x+6$, the x-axis, the lines $x=-2$ and $x=3$

5. Find the Volume created by revolving the region bounded by $y=\sqrt{x}$, $y=0$ & $x=3$;

a. around the x-axis & b. around the y-axis

6. Find the area bounded by the following $y=x^2-1$, $y=-x+2$, $x=0$ & $x=1$.

7. Find the volume created by revolving the region bounded by $y=2x-x^2$ & the x-axis about the line $x=0$.

8. Find the equation of the tangent line to $y=\frac{3x}{x^2+1}$ at $x=1$.

Calc I test #3 Problem Set KEY G. Buthusiem

1. $\int_0^1 3x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \quad \to f(x)=3x \quad [0,1]$

$\Delta x = \frac{1}{n} \quad x_i = a + i\Delta x = \frac{i}{n} \quad f(x_i) = f(i/n) = 3\,i/n$

$\sum f(x_i)\Delta x = \sum_{i=1}^{n} \frac{3i}{n}(1/n) = \frac{3}{n^2}\sum_{i=1}^{n} i = \frac{3}{n^2}\left(\frac{n(n+1)}{2}\right)$

$\lim_{n\to\infty} \sum f(x_i)\Delta x = \lim_{n\to\infty} \frac{3}{n^2}\frac{(n)(n+1)}{2} = 3/2$

2. a. $\int \frac{x^2+2x-6}{x^4}\,dx = \int\left(\frac{x^2}{x^4} + \frac{2x}{x^4} - \frac{6}{x^4}\right)dx = \int (x^{-2} + 2x^{-3} - 6x^{-4})\,dx$

$= \frac{x^{-1}}{-1} + \frac{2x^{-2}}{-2} - \frac{6x^{-3}}{-3} + C = -\frac{1}{x} - \frac{1}{x^2} + \frac{2}{x^3} + C$

b. $\int (5\cos x - 2\sec^2 x)\,dx = 5\int \cos x\,dx - 2\int \sec^2 x\,dx$

$= 5\sin x - 2\tan x + C$

c. $\int 6x^3\sqrt{3x^4+2}\,dx = \frac{6}{12}\int u^{1/2}\,du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$

$u = 3x^4+2 \quad \frac{du}{12} = \cancel{12}x^3\,dx \qquad = \frac{1}{3}(3x^4+1)^{3/2} + C$

d. $\int \frac{x+4}{x^2+8x-7}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+8x-7| +$

$u = x^2+8x-7$

$du = (2x+8)\,dx = \cancel{2}(x+4)\,dx$

$\frac{du}{2}$

e. $\int \sin^4 5x \cos 5x\,dx = \frac{1}{5}\int u^4\,du = \frac{1}{5}\frac{u^5}{5} + C$

$u = \sin 5x \qquad = \frac{1}{25}\sin^5 5x + C$

$du = \cancel{5}\cos 5x\,dx$

$\frac{du}{5}$

3. a. $\int_0^3 (5x^3+3x-1)\,dx = \frac{5x^4}{4} + \frac{3x^2}{2} - x \Big|_0^3 = \frac{5}{4}(3)^4 + \frac{3(3)^2}{2} - 3$

$= 3\left(\frac{5 \cdot 27}{4} + \frac{9}{2} - 1\right) = 3\left(\frac{135 + 18 - 4}{4}\right) = \frac{447}{4}$

b. $\int_0^1 (3x+1)^5\,dx = \frac{1}{3}\int_0^1 u^5\,du = \frac{1}{18}u^6 \Big|_0^1 = \frac{1}{18}(3x+1)\Big|_0^1$

$u = 3x+1$

$du = 3dx$

$\frac{du}{3}$

$= \frac{1}{18}\left(3+1 - (0+1)\right) = \frac{1}{6}$

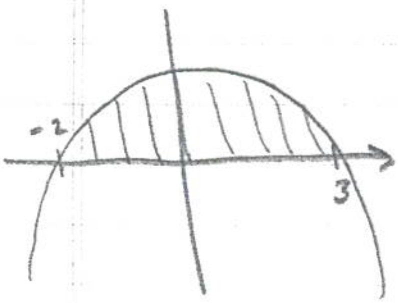
c. $\int_{-\pi/4}^{\pi/4} \sin 2x\,dx = \frac{1}{2}\int_{-\frac{\pi}{4}}^{\pi/4} \sin u\,du = -\frac{1}{2}\cos 2x \Big|_{-\pi/4}^{\pi/4}$

$u = 2x$ $du = 2dx$ $\frac{du}{2}$

$= -\frac{1}{2}\left(\cos(2(\pi/4)) - \cos(2(-\pi/4))\right)$

$= -\frac{1}{2}(0 - 0) = 0$

4. $f(x) = -x^2 + x + 6$ $[-2, 3]$ $\rightarrow$ $f(x)$ has x int at $x=-2$ & $x=3$ and is a downward parabola



$A = \int_{-2}^{3} (-x^2+x+6)\,dx = -\frac{x^3}{3} + \frac{x^2}{2} + 6x \Big|_{-2}^{3}$

$= -\frac{3^3}{3} + \frac{3^2}{2} + 6(3) - \left(-\frac{(-2)^3}{3} + \frac{(-2)^2}{2} + 6(-2)\right)$

$= -\frac{27}{3} + \frac{9}{2} + 18 - \frac{8}{3} - 4/2 + 12 = 19 + \frac{9}{2} - \frac{8}{3} = \boxed{\frac{125}{6}}$