Sum and Difference Identities

To this point we have only looked at identities of a single variable we now need to look at ones with multiple variables.

Remember $\cos\theta$ is "cosine of theta" this is not multiplication.

Sum and Difference Identites for cosine

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

We can use this and the other identities in this section to help us find trig. functions of angles that are not "special" angles.

Ex: Find the exact value of $\cos(105^0)$

Ex: Find the exact value of $\cos\left(\frac{\pi}{12}\right)$

At this point you should have noticed a pattern to the "co" trig. functions as they relate to complimentary angles.

Cofunction Identities

$$\sin\left(\frac{\pi}{2}-u\right)=\cos u \qquad \cos\left(\frac{\pi}{2}-u\right)=\sin u$$
$$\tan\left(\frac{\pi}{2}-u\right)=\cot u \qquad \cot\left(\frac{\pi}{2}-u\right)=\tan u$$
$$\sec\left(\frac{\pi}{2}-u\right)=\csc u \qquad \csc\left(\frac{\pi}{2}-u\right)=\sec u$$

These can easily be seen by looking at examples like $\sin(30^0)=\frac{1}{2}=\cos(60^0)$

Ex: Find the exact value of $\sin\left(15^0\right)$

Sum and Difference Identities for sine

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

Ex: Find the exact value of $\sin(75^0)$

Sum and Difference Identities for tangent

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} \qquad \tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Ex: Find the exact value of $\tan\left(75^0\right)$

Ex: Find the exact values of

a. $\sin\left(\frac{7\pi}{12}\right)$ **b.** $\cos\left(\frac{7\pi}{12}\right)$ **c.** $\tan\left(\frac{7\pi}{12}\right)$ (hint: $\frac{\pi}{4}+\frac{\pi}{3}=\frac{7\pi}{12}$)

Ex: Simply the following expressions

a. $\cos(51^0)\cos\left(6^0\right)+\sin(51^0)\sin(6^0)$

b. $\sin 2\theta\cos 3\theta-\cos 2\theta\sin 3\theta$

Ex: Find the exact value of $\sin(\alpha-\beta)$ given $\sin\alpha=-3/5$ and $\cos\beta=-1/3$ with α in quad IV and β in quad III.

Ex: Verify the identity: $\cos\left(x-90^0\right)=\cos x\tan x$

Ex: Verify the identity: $\frac{\sin(x+y)}{\sin x\cos y}=1-\cot x\tan y$