Solving Trigonometric Equations

Section Objectives: Students will know how to use standard algebraic techniques and inverse trigonometric functions to solve trigonometric equations.

A **solution** to any equation is any value that can be plugged in for the variable(s) that make the equation true, basically making one side equal to the other.

Ex: Find all the solutions to

a. $\cos x = 1$ **b.** $\cos x = 0$ **c.** $\cos x = -1/2$

Solving cosx = *a*

1. If -1 < a < 1 and $a \neq 0$, the solution set is $\{x \mid x = s + 2k\pi\}$, where s = $\cos^{-1} a$

2. The solution set to cosx = 1 is $\{x \mid x = 2k\pi\}$

3. The solution set to cosx = 0 is $\{x \mid x = \pi/2 + k\pi\}$

4. The solution set to cosx = -1 is $\{x \mid x = \pi + 2k\pi\}$

5. If $|a| > 1$, then cosx = a has **NO** solution.

Ex: Find all the solutions to $\sin x = -1/2$

Solving sinx = *a*

1. If -1 < a < 1, $a \neq 0$ and s = $\sin^{-1} a$ the solution set is $\{x \mid x = s + 2k\pi\}$ for s > 0 and $\{x \mid x = \pi - s + 2k\pi\}$ for s < 0.

2. The solution set to sinx = 1 is $\{x \mid x = \pi/2 + k\pi\}$

3. The solution set to sinx = 0 is $\{x \mid x = k\pi\}$

4. The solution set to sinx = -1 is $\{x \mid x = 3\pi/2 + k\pi\}$

5. If $|a| > 1$, then sinx = a has **NO** solution.

Solving tanx = *a*

If a is any real number and s = $\tan^{-1} a$, then the solution set to tanx = a is $\{x \mid x = s + k\pi\}$ for s ≥ 0, and $\{x \mid x = s + \pi + 2k\pi\}$ for s < 0.

Ex: Find all the solutions to $2\cos x - 1 = 0$

Ex: Find all the solutions to $2\sin x = \sqrt{3}$

Ex: Find all the solutions to $\sin 2\theta = \frac{\sqrt{2}}{2}$

Ex: Find all the solutions to $\tan 3x = \sqrt{3}$

Ex: Find all the solutions in the interval $[0, 2\pi]$ **to** $\sin 2x = \sin x$

Ex: Find all the solutions to $6\cos^2 x - 7\cos x + 2 = 0$

Ex: Find all the solutions to $2\sin^2 x + \sin x = 1$

Ex: Find all the solutions in the interval $[0, 360^0]$ that satisfy the equation $\tan 3y + 1 = \sqrt{2}\sec 3y$

Solving Trigonometric equations

1. Know the solutions to sinx = a, cosx = a, tanx = a.
2. Solve an equation involving multiple angles as if it had a single variable.
3. Simplify complicated equations by using identities. Try to get and equations with a single trigonometric function.
4. If possible use factoring and the zero product property.
5. Square each side of the equation if necessary, so you can use identities with squares.