

Right Triangle Trigonometry

Section Objectives: Students will know how to use the fundamental trigonometric identities.

Definitions of Trigonometric Functions of any angle

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y} \quad y \neq 0$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x} \quad x \neq 0$$

$$\tan\theta = \frac{y}{x} \quad x \neq 0 \qquad \cot\theta = \frac{x}{y} \quad y \neq 0$$

Ex: Let $(4, -3)$ be on the terminal side of θ . Find the value of the sine, cosine, and tangent of θ .

Ex: Given $\cos\theta = 3/5$ and $\tan\theta < 0$, find $\sin\theta$.

Right Triangles

Hypotenuse (hyp) - side connecting angle theta and other non right angle

Opposite (opp) - side connecting right angle and other non right angle

Adjacent (adj) - side connecting theta and right angle

Pythagorean Theorem:

$$a^2 + b^2 = c^2 \quad \text{or} \quad (\text{opp})^2 + (\text{adj})^2 = (\text{hyp})^2$$

The Six Trigonometric Functions and Right Triangles

Let θ be an acute angle of a right triangle, the six trig functions of the angle θ are defined as follows:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

opp = the length of the side *opposite* θ

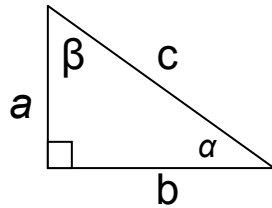
adj = the length of the side *adjacent to* θ

hyp = the length of the *side that does not touch the 90° angle*

Look at right triangles: $45^\circ, 45^\circ, 90^\circ$ and $30^\circ, 60^\circ, 90^\circ$
And evaluate the six trig functions for each theta.

Tip: Since these values are encountered so frequently, you should memorize them, but you already do since you memorized the unit circle.

Ex: Solve the right triangle: $a = 10$ and $c = 12$



Applications Involving Right Triangles

Ex: If the sun is 30° up from the horizon and shining on a tree forming a 50-foot shadow, how tall is the tree?

Ex: If a rope tied to the top of a flagpole is 35 feet long, then what angle is formed by the rope and the ground when the rope is pulled to the ground, 25 feet from the base of the pole?

Ex: A surveyor is standing 115 ft. from the base of the Washington Monument. A surveyor measures the angle of elevation to the top of the monument as 78.3° . How tall is the monument?