Product and Sum Identities

We already learned about the sum and difference identities. Again we can use these to create the product to sum formulas. Think about combining $\sin(A+B)=\sin A\cos B+\cos A\sin B$ and

$\sin(A-B)=\sin A\cos B-\cos A\sin B$

Product to Sum Identities

$$\sin A\cos B=\frac{1}{2}\left[\sin(A+B)+\sin(A-B)\right]$$

$$\sin A\sin B=\frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right]$$

$$\cos A\sin B=\frac{1}{2}\left[\sin(A+B)-\sin(A-B)\right]$$

$$\cos A\cos B=\frac{1}{2}\left[\cos(A-B)+\cos(A+B)\right]$$

Ex: Rewrite each expression:

a. $\sin(4x)\cos(3x)$ **b.** $\sin(\pi/5)\sin(\pi/8)$

Ex: Find the exact value of $\sin(52.5^0)\cos(7.5^0)$

Sum to Product Identities

For these we use the previous identities with the substitution $A + B = x$ and $A - B = y$.

$$\sin x+\sin y=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x-\sin y=2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x-\cos y=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Ex: Rewrite each expression:

a. $\cos(4x)+\sin(3x)$ **b.** $\cos(4x)+\cos(2x)$

Ex: Find the exact value of $\sin(105^0)+\sin(15^0)$

Ex: Verify the identity: $\dfrac{\sin 3x+\sin 5x}{\sin 3x-\sin 5x}=-\dfrac{\tan 4x}{\tan x}$

Ex: Verify the identity: $\dfrac{\cos 5y+\cos 3y}{\cos 5y-\cos 3y}=-\cot 4y\cot y$