Operations with Functions

Section Objectives: Students will know how to find arithmetic combinations and compositions of functions.

Arithmetic Combinations of Functions

Two functions can be combined to create a new function.

Let *f* and *g* be functions with overlapping domains. Then for all *x* common to both domains:

1. $(f + g)(x) = f(x) + g(x)$

2. $(f - g)(x) = f(x) - g(x)$

3. $(f \bullet g)(x) = f(x) \cdot g(x)$

4. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$

Ex: Given $f(x) = x^2 + 2x$ and $g(x) = 2x + 1$, find the following.

a) $f(x) + g(x)$ **b)** $f(x) - g(x)$ **c)** $f(x) \cdot g(x)$ **d)** $\frac{f(x)}{g(x)}$

Tip: Remember to consider the domains of quotients of functions.

Composition of Functions

One way of combining functions is with compositions.

The composition of the function f with the function g is

$$(f \circ g)(x) = f(g(x))$$

The domain of $(f \circ g)$ is the set of all x in the domain of g such that g(x) is in the domain of f.

Composition of functions deals with the order in which you evaluate the functions it is NOT multiplication

Ex: Given $f(x) = x^2 + 2x$ and $g(x) = 2x + 1$, find the following.

(a) $(f \circ g)(x)$ **(b)** $(g \circ f)(x)$ **(c)** $(g \circ f)(3)$

Ex: $h(x) = (x + 1)^2$ Find two functions f and g such that $h(x) = (f \circ g)(x)$.