Law of Sines

Section Objectives: Students will know how to use the computational applications of the **Law of Sines** to solve a variety of problems.

Any triangle without a right angle is called an **oblique triangle**. To solve an **oblique** triangle, we need to be given at least one side and then any other two parts of the triangle. There are four possibilities:

1. One side and any two angles (ASA, AAS)
2. Two sides and a non-included angle (SSA)
3. Two sides and an included angle (SAS)
4. Three sides (SSS)

We can solve the first two cases with the Law of Sines and the second two with the Law of cosines.

The Law of Sines: For any triangle with angles ***A, B,*** and ***C*****,** and opposite sides ***a*****,** ***b*****,** and ***c,*** respectively**,** the following equations are true**:**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

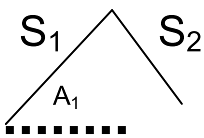
Solving Triangles: The AAS and ASA cases: Two angles and a side of a triangle**,** in any order**,** determine the size and shape of a triangle completely**.** Of course**,** two angles of a triangle determine the 3^{rd}**.** We solve for the remaining 2 parts **(**the unknown sides**)** with the **LAW OF SINES.**

Example 1: Given A **=** 123^0**,** B **=** 41^0**,** and a **=**10 inches**,** find ***c*****:**

Example 2: A triangular plot of land has **interior** **angles** A **=** $95^0$and C **=** 68^0**.** If the side between these angles is **115 yards** long**,** what are the **lengths** of the other two sides**?**

The Ambiguous Case (SSA): While 2 angles and a side of a triangle are always sufficient to determine its size and shape, <u>the same can not be said for 2 sides and an angle</u>**; it depends on where the angle is.** If the <u>angle</u> is included <u>between the 2 sides</u> **(SAS),** then the triangle is uniquely determined up to congruence**.** If the angle is opposite one of the sides **(SSA),** then there might be 1**,** 2**,** or 0 triangles determined**.**

The **SAS case can be solved by using the **LAW OF <u>COSINES</u> (next section),** while the **SSA** case can be solved by the **LAW OF SINES.**



The ambiguous case exists is if $S_1 > S_2$ and A_1 is acute

- If $S_2 > h$ then 2 triangles exist
- If $S_2 = h$ then 1 triangle exists
- If $S_2 < h$ then NO triangles exist

where $h = S_1 \sin A_1$

Example 3: Given $A = 26^0$, $b = 5$ feet, and $a = 21$ feet, find the other side and the two other angles of the triangle.

Because $a > b$, there will be one triangle.

Example 4: Given B **=** 78^0**,** c **=** 12**,** and b **=** 5**,** find angle C**.**

Example 5: Given A **=** 29^0**,** a **=** 6**,** and b **=** 10**,** find B**:**
Since h **=** 10 sin $29^0 \approx 4.85 < 6 < 10$**,** there are two triangles and hence two values for B**,** B_1 and B_2, which are supplementary**:**