Inverse Trigonometric Functions

Section Objectives: Students will know how to evaluate the inverse trigonometric functions and compositions of trigonometric functions

Inverse Sine Function

Because the sine function does not pass the **Horizontal Line Test**, we must restrict its domain in order for its inverse to be a function.

We restrict the domain to the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

1. On the interval $y = sinx$ is increasing
2. On the interval $y = sinx$ takes of the full range of values $-1 \le \sin x \le 1$
3. On the interval $y = sinx$ is one-to-one

This restricted interval of $y = sinx$ has a unique inverse function called the **inverse sine function** denoted by

$$y = \sin^{-1} x \text{ or by } y = \arcsin x.$$

It can be thought of as the angle whose sine is *x*.

Defintion: The inverse sine function is defined by

$$y = \arcsin x \text{ iff } \sin y = x$$

where $-1 \le x \le 1$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

The **domain** of y = arcsinx is [-1,1]

The **range** of y = arcsinx is $\left[-\pi/2, \pi/2\right]$

Ex: Evaluate the following.

a) $\sin^{-1}(-1/2) =$ **b)** $\arcsin(\sqrt{3}/2) =$ **c)** $\sin^{-1}(2) =$

Ex: Sketch y = arcsinx

Definitions of Inverse Functions		
Functions	*Domain*	*Range*
y = arcsinx	-1 ≤ x ≤ 1	-π/2 ≤ y ≤ π/2
y = arccosx	-1 ≤ x ≤ 1	0 ≤ y ≤ π
y = arctanx	-∞≤ x ≤ ∞	-π/2 < y < π/2
y = arccotx	-∞≤ x ≤ ∞	0 < y < π
y = arcsecx	$\lvert x \rvert \ge 1$	0 ≤ y ≤ π, y ≠ π/2
y = arccscx	$\lvert x \rvert \ge 1$	-π/2 ≤ y ≤ π/2, y≠0

Ex: Evaluate the following.

a) $\arccos\left(\sqrt{2}/2\right)$ **b)** $\cos^{-1}(-1)$ **c)** $\arctan 0$ **d)** $\tan^{-1}(-1)$

Compositions of Functions

Remember $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

Inverse properties of trigonometric functions.

1. If $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$, then

$\sin(\sin^{-1} x) = x$ and $\sin^{-1}(\sin y) = y$.

2. If $-1 \le x \le 1$ and $0 \le y \le \pi$, then

$\cos(\cos^{-1} x) = x$ and $\cos^{-1}(\cos y) = y$.

3. If x is a real number and $-\pi/2 \le y \le \pi/2$, then

$\tan(\tan^{-1} x) = x$ and $\tan^{-1}(\tan y) = y$.

The inverse properties do not apply to arbitrary values of *x*

Ex: Evaluate the following.

a. $\tan(\arctan -5)$ **b.** $\arcsin(\sin 5\pi/6)$ **c.** $\cos(\cos^{-1}\pi)$

Ex: Find the exact value of the following

a. $\sin(\arctan(0))$ **b.** $\arcsin(\cos(\pi/6))$ **c.** $\tan(\sec^{-1}(\sqrt{2}))$