

Inverse Functions

Section Objectives: Students will know how to find inverses of functions graphically and algebraically.

Definition of Inverse Function

Let f and g be two functions such that

1. $f(g(x)) = x$ for every x in the domain of g
and

2. $g(f(x)) = x$ for every x in the domain of f

In this case g is the inverse of f and we write $g(x)$ as $f^{-1}(x)$

*This is **NOT** a negative exponent it is just the notation*

The domain of $f(x)$ must be the range of $f^{-1}(x)$ and the range of f must be the domain of $f^{-1}(x)$

Ex: $f(x) = x + 2$ and $g(x) = x - 2$ are inverses because

$$f(g(x)) = f(x - 2) = (x - 2) + 2 = x, \text{ and}$$

$$g(f(x)) = g(x + 2) = (x + 2) - 2 = x$$

The Graph of an Inverse Function

If a point (a,b) is on the graph of $f(x)$ then the point (b,a) is on the graph of $f^{-1}(x)$. This happens because the domain and ranges get reversed.

If we interchanged the x 's and y 's for every point on the graph of a function, the graph would be reflected about the line $y = x$.

Ex: $f(x) = x - 1$ and $f^{-1}(x) = x + 1$

One-to-One Functions

Draw the graph of $y = x^2$. Reflect it about the line $y = x$.

Is this inverse a function.

When is the inverse of a function a function?

This brings us to the definition of a **one-to-one function**:

- A function f is **one-to-one** if each value of the dependent variable corresponds to exactly one value of the independent variable.

A function has an **inverse** *iff* the function is **one-to-one**.

This brings us to the **Horizontal Line Test**:

- If every horizontal line meets the graph of a function in at most one point, then the inverse of this function will be a function.
- If no horizontal line intersects the graph of a function in more than one point, no x -value is matched with more than one y -value.

Ex: Which of the following functions has an inverse function?

(a) $f(x) = |x|$

(b) $g(x) = x^3$

Finding Inverse Functions Algebraically

Steps for finding the inverse of a function:

1. Is the function one-to-one?
2. Replace $f(x)$ with y .
3. Interchange the roles of x and y .
4. Solve this new equation for y .
5. Replace y by $f^{-1}(x)$.

Ex: Find the inverse of each of the following functions.

(a) $f(x) = \frac{5-3x}{2}$

(b) $g(x) = \sqrt[3]{x+1}$

(c) $h(x) = x^2$ for $x \geq 0$