Graphs of Sine and Cosine Functions

Section Objectives: Students will know how to sketch and translate the graphs of sine and cosine functions.

The graph of the sine function is a **sine wave,** a **sinusoidal wave,** or a **sinusoid.**

The sine and cosine functions have domain: $(-\infty,\infty)$ and range: [-1,1].

If y = f(x) is a function and a nonzero constant such as f(x) = f(x + a) for every x in the domain of f, then f is called a **periodic function**. The smallest such positive constant a is the **period (cycle)** of the function.

The period for the sine and cosine functions is 2π.

To develop these graphs we look at **five key points** of the graphs:

$$0,\frac{\pi}{2},\pi,\frac{3\pi}{2},2\pi$$

these represent intercepts, maximums, and minimums.

Amplitude and Period

Look at y = $\boldsymbol{a}$sin x and y = $\boldsymbol{a}$cos x

a creates a vertical stretch or shrink

if $|a|>1$ *stretch*

if $|a|<1$ *shrink*

therefore y = $\boldsymbol{a}$sinx and y = $\boldsymbol{a}$cos x ranges between **–$\boldsymbol{a}$** & **$\boldsymbol{a}$** instead of **-1** and **1**

The **amplitude** of y = **a**sin x and y = **a**cos x represents half the distance between the maximum and the minimum values of the function and is given by

Amplitude = $|a|$

Ex: Use the key points to

Sketch $y=2\sin x$ on the interval $[-2\pi,2\pi]$

Ex: On the same coordinate axes sketch the graphs of

$$y=\frac{1}{2}\cos x \quad \textit{and} \quad y=3\cos x$$

Use the five key points

Remember: ***y = -f(x)*** *is a* ***reflection*** *in the* ***x- axis*** *of the graph y = f(x)*

Ex: Sketch the graph of **y = -3sinx**

Period of Sine and Cosine Functions

Let b be a positive real number.

The period of y = ***a*** sin***b***x and y = ***a*** cos***b***x is $2\pi / b$

if 0< b <1 the period is greater than 2π

if b >1 the period is less than 2π

This will affect your key points

Ex: Sketch the graph of $y = \sin\frac{x}{2}$, $b = \frac{1}{2}$ *so period is* 4π

Translations of Sine and Cosine Curves

For y = ***a*** sin (***b(***x – ***c)***) and y = ***a*** cos (***b(***x – ***c)***), **c** creates a **horizontal translation** (phase shift) of the basic **sine** and **cosine** curves

The left and right endpoints of a one-cycle interval can be determined by solving the equations

b(x – c) = 0 and **b(x – c) =** 2π

This will also change your key points in between the endpoints

Ex: Sketch the graph:

$$y = \frac{1}{2}\sin(x - \frac{\pi}{3})$$

$$y = -3\cos(2\pi x + 4\pi)$$

Vertical Translation

y = ***a*** sin (***b(***x – ***c)***) + ***d*** and y = ***a*** cos (***b(***x – ***c***))+ ***d***

if ***d*** **> 0 move up**

if ***d*** **< 0 move down**

Ex: Sketch the graph of y = **2** + **3** cos **2**x

Remember to look at the key points but since the period is π the key points are over the interval $[0, \pi]$