Graphs of Functions

Section Objectives: Students will know how to use the Vertical Line Test, find zeros of functions, identify intervals on which functions are increasing or decreasing, and identify even and odd functions.

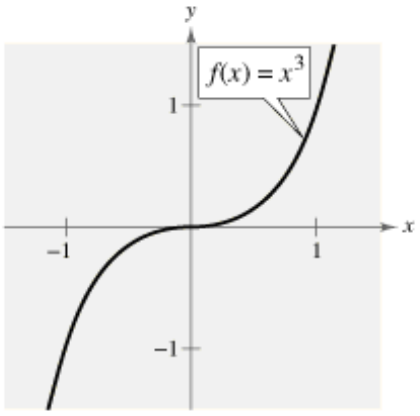
Since $y = f(x)$, graphing functions is no different from graphing equations in two variables.

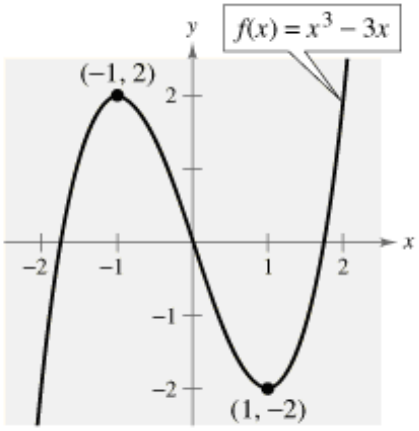
The **graph of a function** *f* is the collection of ordered pairs $(x, f(x))$ such that *x* is in the domain of *f*.

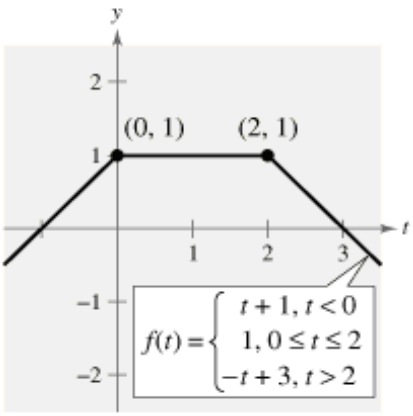
Increasing and Decreasing Functions

1. A function *f* is **increasing** on an interval if, for any x_1 and x_2 in the interval such that $\mathbf{x_1 < x_2}$, $\mathbf{f(x_1) < f(x_2)}$.
2. A function *f* is **decreasing** on an interval if, for any x_1 and x_2 in the interval such that $\mathbf{x_1 < x_2}$, $\mathbf{f(x_1) > f(x_2)}$.
3. A function *f* is **constant** on an interval if, for any x_1 and x_2 in the interval, $\mathbf{f(x_1) = f(x_2)}$.

Find the intervals for which the following functions are increasing and/or decreasing.







Families of Functions

The graph of every **linear function** $f(x) = ax + b$ is a line with **slope** $m = a$ and **y-intercept $(0, b)$**. The graph of a linear function has the following features:

- Domain: All real numbers
- Range: All real numbers
- One intercept at $(0, b)$.
- The graph is increasing if $m > 0$, decreasing if $m < 0$, and constant if $m = 0$.

A **constant function** has the form $f(x) = c$. The graph of a constant function is a horizontal line.

The **identity function** has the form $f(x) = x$. The graph of the identity function has slope 1 and passes through (0, 0). For the identity function, every x-coordinate is the same as its corresponding y-coordinate.

The graph of the **squaring function** $f(x) = x^2$ is a U-shaped curve. The graph has the following features:

- Domain: All real numbers
- Range: All *nonnegative* real numbers
- The function is even
- Intercept at (0, 0)
- Decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$
- Symmetric with respect to the y-axis
- Relative minimum at (0, 0)

The graph of the **cubic function** $f(x) = x^3$ has the following features:

- Domain: All real numbers
- Range: All real numbers
- The function is odd
- Intercept at (0,0)
- The graph is increasing on the interval $(-\infty, \infty)$
- The graph is symmetric with respect to the origin

The graph of the **square root function** $f(x)=\sqrt{x}$ has the following features:

- Domain: All *nonnegative* real numbers
- Range: All *nonnegative* real numbers
- The graph has an intercept at (0,0)
- The graph is increasing on the interval $(0,\infty)$

The graph of the semicircle function $f(x)=\sqrt{r^2-x^2}$ has the following features:

- Domain: all x such that $r^2-x^2>0$
- Range: All *nonnegative* real numbers
- The graph is half a circle

The **greatest integer function** (step function), denoted by $f(x)=[\![x]\!]$, is

[|x|] = the greatest integer less than or equal to x

This is commonly referred to as a step function. It is the type of function telephone companies use to bill us for long distance calls.

Ex: Evaluate the function when x = -2, 5/2, and 4.

f(x) = [|x|] + 4

A **piecewise function** is a function defined by two or more function dependant on specific x values.

Ex:. Sketch the graph of

$$f(x)=\begin{cases}2x+3, & x\le 1\\ -x+4, & x>1\end{cases}$$

The most common piecewise function is the absolute value function

$$f(x)=|x|$$

Common Functions

The eight most common functions are:

- The **constant function**, f(x) = c
- The **identity function**, f(x) = x
- The **absolute value function**, f(x) = $|x|$
- The **square root function**, f(x) = $\sqrt{x}$
- The **quadratic function**, f(x) = x^2
- The **cubic function**, , f(x) = x^3
- The **greatest integer function**, f(x) = [|x|]