Double and Half Angle Identities

We use the sum and difference identities to help prove the double and half angle identities. Think about $\sin 2x = \sin(x+x)$

Double-Angle Indentites

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Remember : $\cos 2x \neq 2\cos x$ no matter how much you want this to work it doesn't!!!

Ex: Find $\sin(60^0)$, $\cos(60^0)$, $\tan(60^0)$ using the double angle identities

Half-Angle Identities

We can use the double angle identities to get the half angle identities. Think about $2\cos^2 x - 1 = \cos 2x$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$$

Ex: Find the exact value of $\cos(-22.5^0)$, $\sin\left(\frac{\pi}{8}\right)$, $\cos(75^0)$, and $\tan(-15^0)$ using the half angle identity.

Ex: Find $\cos(\alpha/2)$ given that $\sin\alpha = 1/3$ and $\pi/2 < \alpha < \pi$

Ex: Verify $\cos 3x = (\cos^3 x - 3\cos x \sin^2 x)$ is an identity

Ex: Verify $\sin^2(x/2)\cos^2(x/2) = \frac{\sin^2 x}{4}$

Ex: Verify $\frac{\sin 4x}{4} = \cos^3 x \sin x - \sin^3 x \cos x$