

Basic Identities

Section Objectives: Students will know how to use fundamental trigonometric identities to evaluate trigonometric functions and simplify trigonometric expressions.

We use trig. identities to:

1. Evaluate trig. functions
2. Simplify trig. expressions
3. Develop additional trig. identities
4. Solve trig. Equations

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1, \quad 1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

The pythagorean identities can also be expressed as radicals

Even/Odd Identities

$$\sin(-x) = -\sin x$$

$$\tan(-x) = -\tan x \quad (\text{odd})$$

$$\cot(-x) = -\cot x$$

$$\csc(-x) = -\csc x \quad (\text{odd})$$

$$\cos(-x) = \cos x$$

$$\sec(-x) = \sec x \quad (\text{even})$$

Ex: If $\sec u = -5/3$ and $\tan u > 0$, find the values of the other five trigonometric functions.

Ex: Write $\sec(x)$ in terms of $\tan(x)$.

Ex: Simplify the following.

- a. $\sin x \cos^2 x - \sin x$ (Hint: Factor and use Pythagorean identity)
b. $\sin t + \cot t \cos t$
c. $\tan t \sin t + \cos t$
d. $\sec(-x) - \sec(x)$

Ex: Rewrite so that it is not a fraction.

$$\frac{1}{1 + \sin x}$$

Ex: Perform the addition and simplify

$$\frac{\sin \theta}{1 + \cos \theta} + \frac{\cos \theta}{\sin \theta}$$

Ex: Factor the following trigonometric expressions.

- a. $\cos^2 x - 1$ b. $\sin^2 u - 3\sin u - 10$
c. $4\tan^2 x + \tan x - 3$ d. $\csc^2 x - \cot x - 3$

Ex: Find an equivalent algebraic expression for the composition

- a. $\cos(\arcsin(x))$ b. $\tan(\arccos(x))$

Ex: Show that this is **NOT** an identity

$$\tan^2 \theta - 1 = \sec^2 \theta$$