Angles: Radian and Degree Measure

Section Objectives: Students will know how to describe an angle and to convert between degree and radian measures.

Trigonometry comes from the Greek word meaning measurement of triangles

1. An **angle** is two rays with the same initial point.
2. The **measure of an angle** is the amount of rotation required to rotate one side, called the **initial side**, to the other side, called the **terminal side**.
3. The shared initial point of the two rays is called the **vertex** of the angle.
4. An angle is in **standard position** if its vertex is at the origin of the rectangular coordinate system and its initial side lies along the positive *x*-axis.
5. If the rotation of an angle is in the **counterclockwise** direction, then the angle is said to be **positive.** If the rotation is **clockwise**, then the angle is **negative.**
6. Two angles in standard position that have the same terminal side are said to be **coterminal.**

Angles are typically labeled with Greek letters

$$\alpha\,(alpha),\quad \beta\ (beta),\ \theta\ (theta)$$

as well as upper case letters A,B, and C

The **measure of an angle** is determined by the **amount of rotation** from the initial side to the terminal side.

Degree Measure

The amount of rotation in an angle with measure one **degree,** denoted by 1^0, is equivalent to the rotation in 1/360 of an entire circle about the vertex.

a **full revolution = 360^0**
a **half of a revolution = 180^0**
a **quarter of a revolution = 90^0**
and so on…..

The most common angles you will see are multiples of 30^0, 45^0, and 60^0

Radian Measure

Another way to measure angles is in **radians**, which is useful in calculus. To define we use a central angle of a circle, one whose vertex is in the center of the circle

Definition:

One **radian** is the measure of a central angle θ that intercepts an arc ***s*** equal in length to the radius ***r*** of a circle.

In general, the radian measure of a central angle θ is obtained be dividing the arc length *s* by *r*, that is $s/r=\theta$ measured in radians.

Since the circumference of a circle is $2\pi r$ the arclength s (of the entire circle) is $2\pi r$. Because $2\pi \approx 6.28$, there are just over six radius lengths in a full circle.

One full revolution has radian measure of 2π

a **half of a revolution** = $\frac{2\pi}{2}=\pi$ radians

a **quarter of a revolution** = $\frac{2\pi}{4}=\frac{\pi}{2}$ radians

a **sixth of a revolution** = $\frac{2\pi}{6}=\frac{\pi}{3}$ radians

and so on…..

We usually do not write the units for radian measure because it is unitless; arc length and radius have the same units.

(if you see something like $\theta=2$ assume its radians unless you see the degree symbol)

Definitions:

acute angles have measure $0<\theta<\frac{\pi}{2}$ or $0^0<\theta<90^0$

obtuse angles have measure $\frac{\pi}{2}<\theta<\pi$ or $90^0<\theta<180^0$

Two angles are **coterminal** if they have the same initial and terminal side.

Some **coterminal** angles are:

$0 \ and \ 2\pi$ or $\frac{\pi}{6} \ and \ \frac{13\pi}{6}$ or $180^0 \ and \ -180^0$

Coterminal angles can be found be adding or subtracting 2π or 360^0 to or from the angle

Ex: Sketch and find coterminal angles for the following: $\frac{3\pi}{2}; -\frac{2\pi}{3}; 135^0$

Two **positive** angle $\alpha \text{ and } \beta$ are **complimentary** if their sum is $\frac{\pi}{2}$ or 90^{0}

Two **positive** angles $\alpha \text{ and } \beta$ are **supplementary** if their sum is π or 180^{0}

Ex: Find the compliment and supplement of $\frac{2\pi}{5}$

Ex: Find the compliment and supplement of 56^{0}

Since 2π radians corresponds to one revolution and 360^{0} is one revolution, degrees and radians are related by the equations

$$360^{0} = 2\pi \text{ radians} \quad \text{and} \quad 180^{0} = \pi \text{ radians}$$

Therefore: $\mathbf{1^{0}} = \frac{\pi}{180}$ radians and **1 radian =** $\frac{180^{0}}{\pi}$

Conversions between Degrees and Radians

1. Convert **Degrees to Radians**: multiply degrees by $\frac{\pi}{180^{0}}$
2. Convert **Radians to Degrees**: multiply radians by $\frac{180^{0}}{\pi}$

Remember: we do not write the units for radian measure

Convert the following degree measures to radian measure:

a) 135^{0} **b)** 540^{0}

Convert the following radian measures to degrees:

a) $\frac{\pi}{30}$ **b)** $\frac{\pi}{5}$

Arc Length:

Because with radian measure $\theta = s/r$, where s is the arc length, $s = r\theta$.

Finding arc length: A circle has radius 4 in, Find the length of the arc intercepted by a central angle of 240^{0}?

(The key is 1^{st} you need radians to use the formula s = r θ then apply the formula)

Linear and Angular Velocity (Speed)

When an object moves at a constant speed in a circular path with a radius of ***r***

The **linear velocity (**v**)** of the object is a measure of how fast the position of the object changes and is given by

$$v = \frac{s}{t} \text{ or } v = \frac{r\theta}{t}$$

where ***t*** is time and ***θ*** is an angle measure in radians.

This is a form of the $\textbf{rate} = \frac{\textbf{distance}}{\textbf{time}}$

The **angular velocity (**ω**,** omega**)** of the object is a measure of how fast the angle of rotation for the object changes and is given by

$$\omega = \frac{\theta}{t}$$

where ***θ*** is an angle measure in radians and ***t*** is time.

Ex: A Ferris wheel with a 50 foot radius make 1.5 revolutions per minute.

- **a.** Find the **angular speed** of the Ferris wheel in radians per minute.
- **b.** Find the **angular speed** of the Ferris wheel in radians per second.
- **c.** Find the **linear speed** of the Ferris wheel in feet per minute.