Volume: The Disk Method and Washer Method

Objective: Find the volume of a solid of revolution using the disk method. Find the volume of a solid of revolution using the washer method. Find the volume of a solid with known cross sections.

If a region in a plane is revolved about a line, the resulting solid is a **solid of revolution**, and the line is called the **axis of revolution**.

The simplest solid is a disk, a rectangle revolved about an axis, and the

Volume of disk = (area of the disk)(width of the disk)

= $\pi R^2 w$ (where R is the radius of the disk)

To relate this to other solids we can approximate the solid using n such disks of width Δx, so

Volume of solid ≈ $\pi \sum_{i=1}^{n} [R(x_i)]^2 \Delta x$

This approximation becomes better if we let the number of discs go to infinity

Volume of solid = $\lim_{n \to 0} \pi \sum_{i=1}^{n} [R(x_i)]^2 \Delta x = \pi \int_a^b [R(x)]^2 dx$

The Disk Method

To find the volume of a solid of revolution with the disk method, use one of the following,

Horizontal Axis of Revolution

$$Volume = V = \pi \int_a^b [R(x)]^2 dx$$

Vertical Axis of Revolution

$$Volume = V = \pi \int_c^d [R(y)]^2 dy$$

Ex: Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = \sqrt{\sin x}$ and the x-axis ($0 \le x \le \pi$) about the x – axis.

Ex: Find the volume of the solid formed by revolving the region bounded by $\mathbf{f(x) = 2 - x^2}$ and g(x) = 1 about the line $\mathbf{y = 1}$.

<u>The Washer Method</u>

The disk method can be extended to cover solids of revolution with holes by replacing the representative disk with a representative washer.

Volume of a washer = $\pi(R^2 - r^2)w$

(where R is the outer radius and r is the inner radius)

Through the same methods as before we get

$$V = \pi \int_a^b \left([R(x)]^2 - [r(x)]^2 \right) dx$$

Ex: Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$ about the x – axis.

Ex: Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis.

Ex: A manufacturer drills a hole through the center of a metal sphere of radius 5 in. The hole has radius 3 in. What is the volume of the resulting metal ring?