# Review of Topics in Algebra and Pre-Calculus

## I. Introduction to Functions

A **function** *f* from a set A to a set B is a relation that assigns to each element *x* in the set A exactly one element *y* in set B. The set A is the domain of *f* and the set B contains the range (or set of outputs).

### Characteristics of a function from set A to set B

**1.** Each element in A must be matched with an element in B.
**2.** Some elements in B may not be matched with any element in A.
**3.** Two or more elements in A may be matched with the same element in B.
**4.** An element in A (the domain) cannot be matched with two different elements in B.

## II. Function Notation

We *name* a function so that it can be referenced. Name the function *f*. Since we say *y* is a function of *x*, replace *y* with *f*(*x*). Now, if we need to know the value of *y* in the function *f* when *x* equals *a*, we just have to write *f*(*a*). This value is read "*f* of *a*."

***Tip:*** There are two concepts that we cannot emphasize too much. One is that *y* = *f*(*x*). The other is that *f* is the *name* of the function, not a variable.

**Ex 1:**. Given $f(x) = x^2 - 4x$ , find the following.

**a)** $f(2)$ **b)** $f(4a)$ **c)** $f(x-2)$ **d)** $f(x+h)$

**Ex 2:** Evaluate when x = -1, 0, 1

$$f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$$

## III. The Domain of a Function

The implied domain of a function is the set of all x such that the corresponding y is a real number. At least for a while, we will only consider three situations,

**1.** Polynomials domain is ($-\infty, \infty$).
**2.** Fractions x cannot be any number in the domain that makes the denominator zero.
**3.** Radicals x if the index is even, then the radicand must be nonnegative.

**Ex:** Find the domains of the following functions.

**a)** f(x) = $x^3$ + 3x + 1 **b)** $f(x) = \frac{1}{x^2 - 1}$ **c)** $f(x) = \sqrt{x-2}$

## IV. The Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

**Ex:** Find the Difference Quotient for

**a)** $f(x) = 3x + 2$

**b)** $f(x) = x^2 + 2x - 1$

**c)** $f(x) = 6x^2 + x$

**Tip:** try to eliminate the h in the denominator

In calculus we see what happens when h $\to$ 0

**<u>Graph of a Function:</u>** the graph of a function *f* consists of all points (x,y) where x is the domain of *f* and *y = f(x)*; that is, all points of the form (x, *f*(x))

**<u>Intercepts:</u>**
The points (if any) where the graph crosses the x axis are called the **x – intercepts**
The point where the graph crosses the y axis is called the **y – intercept**

**How to find the intercepts:**
Y int: let x equal zero
X int: let y or f(x) equal zero

**Ex:** Find the intercepts of $f(x) = -x^2 + x + 2$

**<u>Graphing Parabolas (quadratic functions)</u>**
**1.** Vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$
**2.** Intercepts
**3.** Open up or down

**<u>Power Functions, Polynomials, and Rational functions:</u>**

**Power functions:** any function of the form $\mathbf{f(x) = x^n}$ where n is a real number

**Polynomial functions:** any function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots + a_1 x^1 + a_0 x^0$$

where n is a nonnegative integer and $a_n$ etc are constants

**Rational Functions:** any function of the form $\frac{p(x)}{q(x)}$ where p and q are polynomials

## Vertical Line Test

A curve is a graph of a function if and only if no vertical line intersects the curve more than once.

## Linear Functions

A function whose value changes at a constant rate with respect to its independent variable is called a **Linear Function.**

**Linear functions** are any functions of the form:

$$f(x) = a_1 x + a_0$$

or

$$f(x) = mx + b$$

The graph of a linear function is a **straight line**.

The steepness of a line can be measured by the **slope** represented whit the letter **m**.

$$m = \frac{change\,in\,y}{change\,in\,x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of the line determines the direction of the line.

## Slope Intercept Form of the Equation of a Line

Given the slope of a line m and the y intercept (0,b) the equation of the line can be found using the equation:

$$y = mx + b$$

## Point Slope Form of the Equation of a Line

Given the slope of a line m and a point on a line ($x_1$,$y_1$) the equation of the line can be found using the equation:

$$y - y_1 = m(x - x_1)$$

## Parallel and Perpendicular Lines

Let $\mathbf{m_1}$ and $\mathbf{m_2}$ be the slopes of the nonvertical lines $\mathbf{L_1}$ and $\mathbf{L_2}$ then:

$\mathbf{L_1}$ and $\mathbf{L_2}$ are parallel if and only if $\mathbf{m_1 = m_2}$

$\mathbf{L_1}$ and $\mathbf{L_2}$ are perpendicular if and only if $\mathbf{m_2} = \frac{-}{\;}$

## Exponential and Logarithmic Functions

The **exponential function** ***f(x)* with base *a*** is denoted by

$$f(x) = a^x$$

where $a > 0$, $a \neq 1$, and $x$ is any real number.

Since the exponential function $\mathbf{f(x) = a^x}$ is one-to-one, its inverse is a function. The function given by

$$f(x) = \log_a x$$

where $x > 0$, $a > 0$, and $a \neq 1$

is called the **logarithmic function with base *a*.**

Furthermore, the logarithmic function with base *a* is the inverse of the exponential function with base *a*; thus

$y = \log_a x$ **if and only if** $x = a^y$

(the two statements are equivalent)

## Properties of a logarithmic function

1. $\log_a 1 = 0$ because $a^0 = 1$
2. $\log_a a = 1$ because $a^1 = a$
3. $\log_a a^x = x$ and $a^{\log_a x} = x$ *Inverse Property*
4. If $\log_a x = \log_a y$, then $x = y$. *One to one Property*
5. $\log_a(uv) = \log_a u + \log_a v$
6. $\ln(u/v) = \ln u - \ln v$
7. $\log_a u^n = n \log_a u$

## Trigonometric Functions

Let $\theta$ be an acute angle of a right triangle, the six trig functions of the angle $\theta$ are defined as follows:

$$\sin\theta = \frac{opp}{hyp} \qquad \csc\theta = \frac{hyp}{opp}$$

$$\cos\theta = \frac{adj}{hyp} \qquad \sec\theta = \frac{hyp}{adj}$$

$$\tan\theta = \frac{opp}{adj} \qquad \cot\theta = \frac{adj}{opp}$$

Remember your special triangles!!!!

Let $\theta$ be an angle in standard position with ($x$, $y$) a point on the terminal side of $\theta$ and $r = \sqrt{x^2 + y^2} \neq 0$.

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y} \quad y \neq 0$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x} \quad x \neq 0$$

$$\tan\theta = \frac{y}{x} \quad x \neq 0 \qquad \cot\theta = \frac{x}{y} \quad y \neq 0$$

## Trigonometric Identities

### Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Second, from the original definitions and the reciprocal identities, we have the **Quotient Identities**

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \ , \ \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Finally, from the Pythagorean Theorem, $(opp)^2 + (adj)^2 = (hyp)^2$, and dividing both sides of the equation by $(hyp)^2$, we have the **Pythagorean Identities.**

$$\sin^2\theta + \cos^2\theta = 1 \quad , \quad 1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Note: $(\sin\theta)^2 = \sin^2\theta \ldots etc.$