Partial Fractions

Objective: Understand the concept of partial fraction decomposition.

In algebra you learned how to add fractions that involved polynomials:

$$\frac{1}{x+2}+\frac{1}{x+3}=\frac{2x+5}{(x+2)(x+3)}$$

Now we can look at pulling the last fraction apart into the sum of the first two fractions. This process of going backwards is called **partial fraction decomposition**.

Decomposition of N(x)/D(x) into Partial Fractions

1. Divide if improper: If N(x)/D(x) is an improper fraction (*the numerator has a higher degree*) divide the denominator into the numerator to obtain

$$\frac{N(x)}{D(x)}=(a\ polynomial)+\frac{N_1(x)}{D(x)}$$

where the degree of $N_1(x)$ is less than the degree of D(x). You are going to decompose this new fraction.

2. Factor the denominator: Completely factor the denominator into factors of the form

$\mathbf{(px + q)^m}$ and $\mathbf{(ax^2 + bx + c)^n}$

(Linear) *(Quadratic)*

3. Linear factors: For each factor of the $\mathbf{(px + q)^m}$, the partial fraction decomposition must include the following sum of m fractions.

$$\frac{A_1}{(px+q)}+\frac{A_2}{(px+q)^2}+\ldots+\frac{A_m}{(px+q)^m}$$

4. Quadratic Factors: For each factor of the form $\mathbf{(ax^2 + bx + c)^n}$, partial fraction decomposition must include the following sum of n factors.

$$\frac{B_1x+C_1}{ax^2+bx+c}+\frac{B_2x+C_2}{(ax^2+bx+c)^2}+\ldots+\frac{B_nx+C_n}{(ax^2+bx+c)^n}$$

Ex: Write the partial fraction decomposition for

a. $\frac{1}{x^2-5x+6}$ **b.** $\frac{5x^2+20x+6}{x^3+2x^2+x}$ **c.** $\frac{3x^2+x}{(x-2)(x^2+3)}$ **d.** $\frac{8x^3+13x}{\left(x^2+2\right)^2}$

Guidelines for solving the Basic Equation

Linear Factors

1. Substitute the roots of the distinct linear factors into the basic equation.

2. For repeated linear factors, use the coefficients determined in guideline 1 to rewrite the basic equation. Then sub other convenient values of x and solve for remaining coefficients.

Quadratic Factors

1. Expand the basic equation

2. Collect terms according to powers of x.

3. Equate the coefficients of like powers to obtain a system of linear equations involving A, B, C, and so on.

4. Solve the system of linear equations.