Linear Equations in Two Variables

Section Objectives: Students will know how to find the slopes of lines and use slope to write and graph linear equations in two variables.

Equations of the form ***Ax+By+C=0*** are called **linear equations in two variables.** They are called *linear* because their graphs are lines.

The **slope** of a line is a measure of its inclination or steepness.

Finding the Slope of a Line

The slope of a line is the ratio of the change in *y* to the change in *x.* In addition, if we know two points on the line, (x_1, y_1) and (x_2, y_2), then the change in *y* is $y_2 - y_1$ and the change in *x* is $x_2 - x_1$. Therefore, the slope ***m*** of a non-vertical line through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{rise}{run}$$

Ex: Find the slope of the line through each pair of points.

a) (3, -7) and (-4, 2) **b)** (2, -9) and (-6, -9)

c) (5, 4) and (5, -8)

Lines can have positive slope, negative slope, slope of zero, and undefined slope.

Ex: Graph the line with slope m= ¾ and through point (-2, 1)

Writing Linear Equations in Two Variables

If a line contains a point (x_1, y_1) and has a **slope = m**, then the equation of the line can be found by

$$\mathbf{y - y_1 = m(x - x_1)}$$

This is the **point-slope form** of the equation of the line.

Ex: Find the equation of the line with slope 4 that passes through the point (-6, 2).

Ex: Find the equation of the line that passes through the points (5, 1) and (-1, 3).

If a line has **slope = *m*** and ***y*-intercept (0, *b*),** then the equation of the line can be found by

$$y = mx + b$$

This form is called **slope-intercept form**.

Ex: Determine the equation of the line with y – intercept = -2 and a slope of ¾ .

Special forms of linear equations.

1. An equation of the vertical line through any point with an x-coordinate of a is **x = a**.

2. An equation of the horizontal line through any point with a y-coordinate of b is **y = b**.

3. The **general form** (or standard form) of a linear equation is **Ax + By + C = 0**.

Parallel and Perpendicular Lines

1. Two distinct nonvertical lines are parallel if and only if their slopes are equal. That is, $\mathbf{m_1 = m_2}$.

2. Two nonvertical lines are perpendicular if and only if their slopes are negative reciprocals of each other. That is, $\mathbf{m_1 * m_2 = -1}$

Ex: Find the general form of the equation of the line that passes through the point **(1, -3)** and is **(a) parallel to** and **(b) perpendicular to** the line given by **2x + 3y = 1**.

Ex: Determine the equation of the line with x – intercept = -2 and **perpendicular** to **7x = y – 12**.