Inverse Transforms

If F(s) represents the Laplace transform of a function f(t), then we say f(t) is the **inverse Laplace transform** of F(s), we write

$$f(t) = \mathscr{L}^{-1}\{F(s)\}$$

If $\mathscr{L}\{t\} = \frac{1}{s^2}$ then $t = \mathscr{L}^{-1}\left\{\frac{1}{s^2}\right\}$

If $\mathscr{L}\{e^{-3t}\} = \frac{1}{s+3}$ then $e^{-3t} = \mathscr{L}^{-1}\left\{\frac{1}{s+3}\right\}$

Some Inverse Transforms

a. $1 = \mathscr{L}^{-1}\left\{\frac{1}{s}\right\}$

b. $t^n = \mathscr{L}^{-1}\left\{\frac{n!}{s^{n+1}}\right\},\ n = 1,2,3\ldots$

c. $\sin kt = \mathscr{L}^{-1}\left\{\frac{k}{s^2+k^2}\right\}$

d. $\cos kt = \mathscr{L}^{-1}\left\{\frac{s}{s^2+k^2}\right\}$

e. $e^{at} = \mathscr{L}^{-1}\left\{\frac{1}{s-a}\right\}$

f. $\sinh kt = \mathscr{L}^{-1}\left\{\frac{k}{s^2-k^2}\right\}$

g. $\cosh kt = \mathscr{L}^{-1}\left\{\frac{s}{s^2-k^2}\right\}$

Ex: Evaluate: $\mathscr{L}^{-1}\left\{\frac{1}{s^5}\right\}$ and $\mathscr{L}^{-1}\left\{\frac{1}{s^2+7}\right\}$

$\mathscr{L}^{-1}$ **is a Linear Transform**: the inverse Laplace transform is also linear that is for constants α and β

$$\mathscr{L}^{-1}\{\alpha F(s) + \beta G(s)\} = \alpha\mathscr{L}^{-1}\{F(s)\} + \beta\mathscr{L}^{-1}\{G(s)\}$$

Ex: Evaluate $\mathscr{L}^{-1}\left\{\frac{-2s+6}{s^2+4}\right\}$

Ex: Evaluate $\mathscr{L}^{-1}\left\{\frac{1}{(s-1)(s+2)(s+4)}\right\}$

Ex: Evaluate $\mathscr{L}^{-1}\left\{\frac{s+1}{s^2(s+2)^3}\right\}$

Ex: Evaluate $\mathscr{L}^{-1}\left\{\frac{3s-2}{s^3(s^2+4)}\right\}$