Finding Limits Graphically and Numerically

Objective: Estimate a limit using a numerical or graphical approach. Learn different ways that a limit can fail to exist. Study and use a formal definition of limit.

An Intro to Limits

Sketch to graph of

$$f(x) = \frac{x^3 - 1}{x - 1}, \ x \neq 1$$

The graph is a parabola with a hole at (1,3)

Although x can not equal 1 for this function you can see what happens to f(x) as x approaches 1 from **both directions**. The notation used is

$$\lim_{x \to c} f(x) = or \lim_{x \to 1} f(x) =$$

This table shows us what is happing in the graph as well as the limit

X	0.75	0.9	0.99	0.999	1	1.001	1.01	1.1	1.25
f(x)	2.313	2.710	2.970	2.997	?	3.003	3.030	3.310	3.813

Then we can say $\lim_{x \to 1} f(x) = 3$

The limit must be the same from both directions!!!

Three pronged approach to problem solving (finding limits)

- 1. Numerical approach Construct a table of values
- 2. Graphical approach Draw a graph by hand or using technology
- 3. Analytic approach Use algebra or calculus

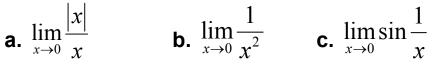
Ex: Find the limit of f(x) as x approaches 2 where f is defined as

$$f(x) = \begin{cases} 1, & x \neq 2\\ 0, & x = 2 \end{cases}$$

Common Types of Behavior Associated with Nonexistence of a Limit:

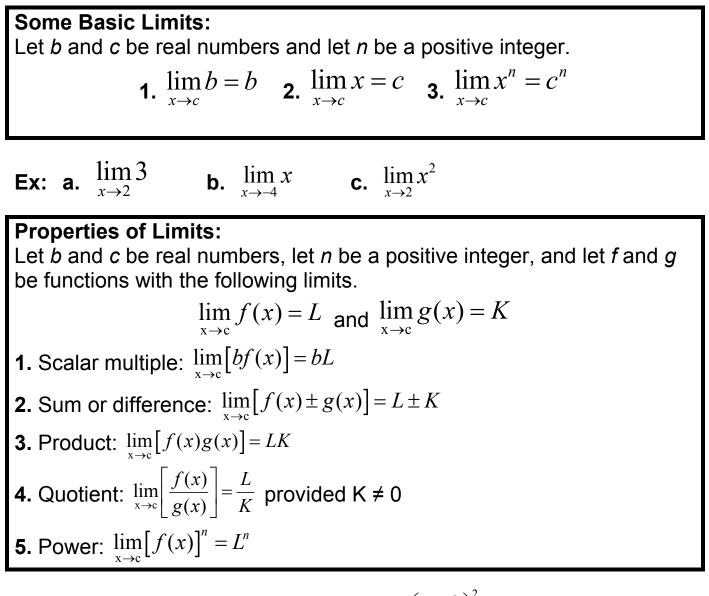
- **1.** f(x) approaches a different number from the right side of c that is approaches from the left side.
- **2.** f(x) increases or decreases without bound as x approaches c.
- **3.** f(x) oscillates between two fixed values as x approaches c.

Some examples of limits that fail to exist



Evaluating Limits Analytically

Objective: Evaluate a limit using properties of limits. Develop and use a strategy for finding limits. Evaluate a limit using dividing out and rationalizing techniques. Evaluate a limit using the Squeeze Theorem.



Ex: a.
$$\lim_{x \to 3} (4x^2 - 2x + 3)(x - 1)$$
 b. $\lim_{x \to -2} \left(\frac{x + 3}{x - 2}\right)^2$

Limits of Polynomials and Rational Functions: If p(x) and q(x) are polynomials and c is a real number, then $\lim_{x \to c} p(x) = p(c) \text{ and } \lim_{x \to c} \frac{p(x)}{q(x)} = \frac{p(c)}{q(c)} \text{ if } q(c) \neq 0$ The Limit of a Function Containing a Radical: Let *n* be a positive integer. The following limit is valid for all *c* if *n* is odd, and is valid for c > 0 if *n* is even.

 $\lim_{x \to c} \sqrt[n]{x} = \sqrt[n]{c}$

Limits of Trigonometric Functions
Let c be a real number in the domain of the given trigonometric function.1. $\lim_{x \to c} \sin x = \sin c$
 $x \to c$ 2. $\lim_{x \to c} \cos x = \cos c$ 3. $\lim_{x \to c} \tan x = \tan c$
 $x \to c$ 4. $\lim_{x \to c} \cot x = \cot c$ 5. $\lim_{x \to c} \sec x = \sec c$
 $x \to c$ 6. $\lim_{x \to c} \csc x = \csc c$

Ex: a. $\lim_{x \to \frac{\pi}{4}} \tan x = \lim_{x \to \pi} (x \sin x) =$ **c.** $\lim_{x \to 0} \cos^2 x =$

Functions That Agree at All But One Point:

Let c be a real number and let f(x) = g(x) for all $x \neq c$ in an open interval containing c. If the limit of g(x) as x approaches c exists, then the limit of f(x) also exists and

$$\lim_{x \to c} f(x) = \lim_{x \to c} g(x)$$

Ex: a. $\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$ b. $\lim_{x \to 3} \frac{3 - x}{x^2 - 9}$

A Strategy for Finding Limits:

- **1.** Learn to recognize which limits can be evaluated by direct substitution
- If the limit of f(x) as x approaches c cannot be evaluated by direct substitution, try to find a function g that agrees with f for all x other than x = c.
- 3. Apply the above theorem to conclude analytically that

 $\lim_{x \to c} f(x) = \lim_{x \to c} g(x) = g(c)$

4. Use a graph or table to reinforce your conclusion

Ex: a. $\lim_{x \to -3} \frac{x^2 + x - 6}{x - 3}$	b. $\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x}$						
Two Special Trigonometric Limits:							
$1. \lim_{x \to 0} \frac{\sin x}{x} = 1$	2. $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$						
Ex: a. $\lim_{x \to 0} \frac{\tan x}{x}$	b. $\lim_{x \to 0} \frac{\sin 4x}{x}$						