Calc III Final Exam Review **G.Buthusiem**

1. Find the equation of the plane through points (2,1,1), (0,4,1), and (-2,1,4).

2. Find the velocity and position vectors of a particle with $\mathbf{a}(t) = \langle 0, t, t \rangle$ and with given initial conditions $\mathbf{v}(1) = 5\mathbf{j}$ and $\mathbf{r}(1) = 0$.

3. Find $\dfrac{\partial z}{\partial y}$ implicitly for $xz + yz + xy = 0$

4. Given $f(x, y) = x^2 + 3y^2$ Find the derivative in the direction of maximum increase at point (1,0,1).

5. Find the equation of the tangent plane and normal line to the surface $xyz = 12$ at (2,-2,-3).

6. Evaluate the double integral by converting to polar coordinates:

$$\iint_R \sin\sqrt{x^2 + y^2}\, dA \text{ where } R = \left\{(x, y) \middle| 0 \le x \le 2, 0 \le y \le \sqrt{4 - x^2}\right\}$$

7. Is $\mathbf{F}(x, y) = \langle 2xy, x^2 \rangle$ conservative? If so prove it and find the potential function f such that $\nabla f = \mathbf{F}$, then integrate **F** over the curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j},\ 0 \le t \le 1$.

8. Evaluate the iterated integral: $\int_0^9 \int_0^{y/3} \int_0^{\sqrt{y^2 - 9x^2}} z\,dz\,dx\,dy$

9. Green's Theorem.

Calc III Final Exam Review G. Bo[illegible]

1. $(2,1,1), (0,4,1), (-2,1,4)$
$\quad P \qquad\qquad Q \qquad\qquad R$

$\vec{PQ} = \langle -2, 3, 0 \rangle \quad \vec{PR} = \langle -4, 0, 3 \rangle$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ -2 & 3 & 0 \\ -4 & 0 & 3 \end{vmatrix} = 9i + 6j + 12k$$ = normal vector to plane

use $P: (2,1,1)$ & $\vec{n} = \langle 9, 6, 12 \rangle$

$9(x-2) + 6(y-1) + 12(z-1) = 0$ plane through three points

2. $a(t) = \langle 0, t, t \rangle$

$\int a(t)\,dt = v(t) = \langle C_1, \frac{t^2}{2} + C_2, \frac{t^2}{2} + C_3 \rangle$

$v(1) = 5j \quad \therefore \quad C_1 = 0, \ \frac{1}{2} + C_2 = 5, \ \frac{1}{2} + C_3 = 0$

$C_2 = 9/2 \qquad C_3 = -1/2$

$\therefore v(t) = \langle 0, t^2/2 + 9/2, t^2/2 - 1/2 \rangle$

$\int v(t)\,dt = r(t) = \langle C_4, \frac{t^3}{6} + \frac{9}{2}t + C_5, \frac{t^3}{6} - \frac{1}{2}t + C_6 \rangle$

$r(1) = 0 \quad \therefore C_4 = 0, \ \frac{1}{6} + \frac{9}{2} + C_5 = 0, \ \frac{1}{6} - \frac{1}{2} + C_6 = 0$

$C_5 = -\frac{14}{3} \qquad C_6 = 1/3$

$\therefore r(t) = \langle 0, \frac{t^3}{6} + \frac{9}{2}t - \frac{14}{3}, \frac{t^3}{6} - \frac{1}{2}t + \frac{1}{3} \rangle$

3. $xz + yz + xy = 0$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{z+x}{x+y}$$

4. $f(x,y) = x^2 + 3y^2 \quad (1,0,1)$

$f_x = 2x \quad f_y = 6y \quad \nabla f = \langle 2x, 6y \rangle$

$\nabla f_{(1,0,1)} = \langle 2, 0 \rangle$ not unit vector

$\nabla f_0 = \frac{1}{2} \langle 2, 0 \rangle = \langle 1, 0 \rangle$

$D_{\nabla f_0} f(x,y) = \langle 2x, 6y \rangle \cdot \langle 1, 0 \rangle = 2x$

at $(1,0,1) = 2$

5. $xyz = 12 \quad (2,-2,-3)$

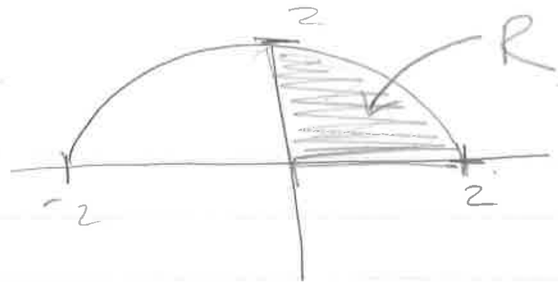
$f_x = yz \quad f_y = xz \quad f_z = xy$

at $(2,-2,-3)$ $f_x = 6 \quad f_y = -6 \quad f_z = -4$

Tangent plane: $6(x-2) - 6(y+2) - 4(z+3) = 0$

Normal line: $x = 2 + 6t$, $y = -2 - 6t$, $z = -3 - 4t$

6. $\iint \sin\sqrt{x^2+y^2}\,dA$



$= \int_0^{\pi/2}\int_0^2 (\sin r) r\,dr\,d\theta$ use IBP

$u = r \quad dv = \sin r\,dr$

$du = dr \quad v = -\cos r$

$\int (\sin r) r\,dr = -r\cos r + \int \cos r\,dr$

$= -r\cos r + \sin r$

$\int_0^{\pi/2} (-r\cos r + \sin r)\Big|_0^2 \,d\theta = \frac{\pi}{2}(-2\cos 2 + \sin 2)$

7. $F(x,y) = \langle \underbrace{2xy}_{F_1}, \underbrace{x^2}_{F_2} \rangle$

$\frac{\partial F_1}{\partial y} = 2x = \frac{\partial F_2}{\partial x}$ conservative

$f(x,y) = \int 2xy\,dx = \int x^2\,dy$

$= x^2y + g(y) = x^2 + h(x)$

$\therefore g(y) = h(x) =$ constant

$f(x,y) = x^2y + C$

$\int F(x,y) = x^2y \Big|_{(0,0)}^{(1,1)} = 1 - 0 = 1$

8. $$\int_0^9 \int_0^{y/3} \int_0^{\sqrt{y^2-9x^2}} z \, dz\, dx\, dy$$

$$= \int_0^9 \int_0^{y/3} \frac{z^2}{2} \Big|_0^{\sqrt{y^2-9x^2}} dx\, dy$$

$$= \int_0^9 \int_0^{y/3} \frac{(y^2-9x^2)}{2} \, dx\, dy$$

$$= \int_0^9 \left(\frac{xy^2}{2} - \frac{3x^3}{2}\right)\Big|_0^{y/3} dy$$

$$= \int_0^9 \left(\frac{y^3}{6} - \frac{y^3}{18}\right) dy = \int_0^9 \frac{y^3}{9} dy = \frac{y^4}{36}\Big|_0^9$$

$$= \frac{9^4}{36} = \frac{729}{4}$$