<u>Conic Sections</u>

<u>Parabola</u>

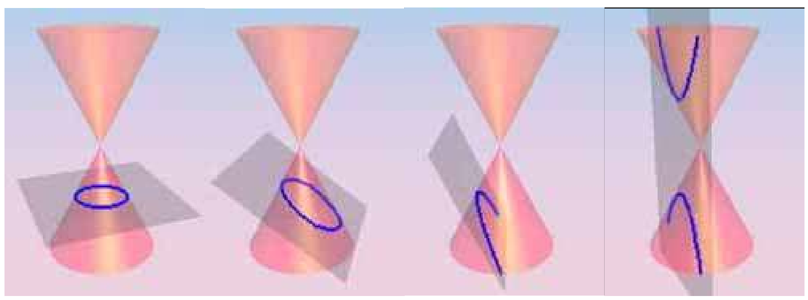
Objective: Define conic section, parabola, draw a parabola, standard equations and their graphs

The curves created by intersecting a double napped right circular cone with a plane are called <u>**conic sections**</u>.

If the plane cuts clear through one nappe and is perpendicular to the axis of the cone the conic is called a <u>**circle**</u> and an <u>**ellipse**</u> if it's not perpendicular.

If the plane cuts only one nappe but does not cut clear through the cone then the conic is a <u>**parabola**</u>.

If the plane cut through both nappes, but not through the vertex the conic is called a <u>**hyperbola**</u>.



Distance is involved in all the definitions of conics so we need the **distance formula**:

Given two points (x_1, y_1) and (x_2, y_2): $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Ex: Find the distance between points (3,4) and (-2,5).

<u>Defintion of Parabola:</u>

A **parabola** is the set of all points P*(x, y)* in a plane whose distance to a fixed point, called the **focus**, equals its distance to a fixed line, called the **directrix**.

The line through the focus perpendicular to the directrix is called the **axis of symmetry**, and the point on the axis of symmetry halfway between the directrix and the focus is called the **vertex**.

Equations of a parabola with vertex (0,0)

$y^2 = 4ax$	$x^2 = 4ay$
Vertex: (0,0)	Vertex: (0,0)
Focus: (a,0)	Focus: (0,a)
Directrix: x = -a	Directrix: y = -a
Symmetric with respect to x-axis	Symmetric with respect to y-axis
Axis of symmetry the x-axis	Axis of symmetry the y-axis

Ex: Locate the focus and directrix and sketch the graph of $y^2 = 24x$

Ex: Locate the focus and directrix and sketch the graph of $x^2 = 32y$

Ex: a. Find the equation of the parabola having the origin as its vertex, y-axis as its axis of symmetry, and (-10,-5) on its graph.
b. Find the coordinates of its focus and the equation of its directrix.

Ex: a. Find the equation of the parabola having the origin as its vertex, y-axis as its axis of symmetry, and (4,2) on its graph.
b. Find the coordinates of its focus and the equation of its directrix.

Circles and Ellipse

Objective: Definition of circle and Ellipse, Draw a circle and ellipse standard equations

Definiton of a circle:

The set of all points P in a plane that are equidistant from a fixed point called the **center**. The fixed distance is the **radius**.

Equation of a circle centered at (0,0)

An equation for the circle with its center at (0, 0) and a radius of *r* is

$$x^2 + y^2 = r^2$$

Ex: Graph and find the standard form of the equation of the circle with center at (0, 0) and radius 4.

Ex: Find the center and radius of the circle with equation $x^2 + y^2 = 10$

Definition of an ellipse:

An **ellipse** is the set of all points *P* in a plane such that the **SUM** of the distances from *P* to two fixed points, F_1 and F_2, (**foci**), is constant.

The line through the foci intersects the ellipse at two points called the **vertices**. . The chord joining the vertices is the **major axis**, and the midpoint is the center of the ellipse. The chord perpendicular to the major axis is the **minor axis**.

Equations of an ellipse with center (*0,0*):

An equation for an ellipse with center (0,0), major axis length is 2a, and minor axis length is 2b where 0 < b < a, is

Horizontal Major Axis:	Vertical Major Axis:
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

In each case:

$a^2 - b^2 = c^2 \quad (\pm c = foci)$

THE MAJOR AXIS IS ALWAYS LONGER THAN THE MINOR AXIS

Ex: Graph and find the standard form of the equation of the ellipse centered at the origin with horizontal major axis = 14 and minor axis = 10.

Ex: Find the coordinates of the foci, find the lengths of the minor and major axis and graph: $\frac{x^2}{4}+\frac{y^2}{9}=1$

Ex: Find the coordinates of the foci, find the lengths of the minor and major axis and graph: $16x^2+9y^2=144$

Hyperbola

Objective: Definition of hyperbola, draw a hyperbola, equation of a hyperbola

Definition of a Hyperbola:

A **hyperbola** is the set of points *P(x, y)* in a plane such that the ABSOLUTE VALUE of the DIFFERENCE between the distances from *P* to two fixed points, F_1 and F_2, called the foci, is a constant.

The intersection points of the line through the foci and the two branches of the hyperbola are called **vertices**, each is called a **vertex**.

The segment between the two vertices is called the **transverse axis**. The midpoint of the transverse axis is the **center**.

The line perpendicular to the transverse axis through the center is the **conjugate axis**.

Equation of a hyperbola with center (*0,0*):

Horizontal Transverse Axis:	Vertical Transverse Axis:
$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$
Asymptotes $y=\pm\frac{b}{a}x$	Asymptotes $y=\pm\frac{a}{b}x$

In each case:

$a^2+b^2=c^2$ $(\pm a = vertices, \pm c = foci)$

The length of the **TRANSVERSE** axis is *2a*

The length of the **CONJUGATE** axis is *2b*.

Ex: Find the coordinates of the foci, find the lengths of the transverse and conjugate axes, find the equations of the asymptotes, and graph the following equation:

$$x^2-9y^2=9$$

Ex: Find the coordinates of the foci, find the lengths of the transverse and conjugate axes, find the equations of the asymptotes, and graph the following equation:

$$4y^2-25x^2=100$$

Ex: Find the equation of the hyperbola with vertical transverse axis centered at the origin with length of the transverse axis of 16, length of the conjugate axis is 24.

Conic Sections NOT Centered at the Origin

Objective: Identify Conics

Identifying Conics

The graph of an equation of the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

is
1. A hyperbola if $B^2 - 4AC > 0$
2. A parabola if $B^2 - 4AC = 0$
3. An ellipse if $B^2 - 4AC < 0$

Standard equations of a parabola with vertex (*h*, *k*):

Vertical Parabola:	**Horizontal Parabola :**
4a(y – k) = (x – h)^2	**4a(x – h) = (y – k)^2**
a > 0: opens upward	**a > 0: opens right**
a < 0: opens downward	**a < 0: opens left**
focus: (*h*, *k* + *a*)	**focus: (*h* + *a*, *k*)**
horizontal directrix: y = k – a	**vertical directrix: x = h – a**
Vertical axis of symmetry: x = h	**Horizontalaxis of symmetry: y = k**

Ex: Find the focus and directrix of $16(x-3) = (y+5)^2$

Ex: Find the vertex, focus, and directrix of $y^2 - 6y - 4x + 1 = 0$.

(you will need to know how to complete the square)

Standard equation of a circle with center (h,k):

An equation for the circle with its center at (*h*, *k*) and a radius of *r* is

$$(x - h)^2 + (y - k)^2 = r^2$$

Ex: Graph and find the standard form of the equation of the circle with center at (2, -5) and radius 4.

Ex: Find the center and radius of the circle with equation

$$x^2 + y^2 - 10x + 6y + 30 = 0$$

Standard equations of an ellipse (*h*, *k*):

An equation for an ellipse with center (h,k), major axis length is 2a, and minor axis length is2b where 0 < b < a, is

Horizontal Major Axis:	Vertical Major Axis:
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$

$a^2-b^2=c^2$

Ex: Find the center, vertices, and foci of the ellipse with equation

$$4x^2+y^2-8x+4y-8=0$$

Ex: Identify the conic $x^2-6x+2y^2+4y+11=0$

Standard equation of a hyperbola with center (*h*, *k*):

Horizontal Tranverse Axis:	Vertical Transverse Axis:
$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$

$a^2+b^2=c^2$

The length of the TRANSVERSE axis is *2a*

The length of the CONJUGATE axis is *2b*.

Ex: Find the equation of the hyperbola with vertices on the line x = - 4, conjugate axis on the line y = 3, length of the transverse axis = 4, and length of the conjugate axis = 6.

Ex: Identify the conic $16x^2-25y^2-160x=0$