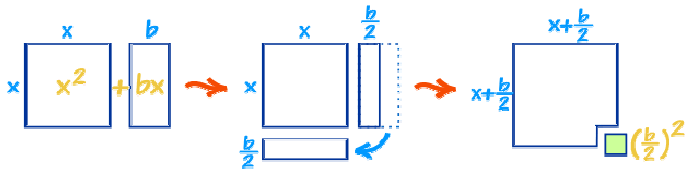
Completing the Square

Say we have a simple expression like $x^2 + bx$. Having x twice in the same expression can make life hard. What can we do?

Well, with a little inspiration from Geometry we can convert it, like this:



As you can see $x^2 + bx$ can be rearranged ***nearly*** into a square ...

... and we can **complete the square** with $(b/2)^2$

In Algebra it looks like this:

$$x^2 + bx \quad + (b/2)^2 \quad = \quad (x+b/2)^2$$

"Complete the Square"

So, by adding $(b/2)^2$ we can complete the square.

And $(x+b/2)^2$ has x only **once**, which is easier to use.

Keeping the Balance

Now ... you can't just ***add*** $(b/2)^2$ without also ***subtracting*** it too! Otherwise the whole value would change.

So I will show you how to do it properly with an example:

Start with: $x^2 + 6x + 7$

("b" is 6 in this case)

Complete the Square: $x^2 + 6x + \left(\frac{6}{2}\right)^2 + 7 - \left(\frac{6}{2}\right)^2$

*Also **subtract** the new term*

Simplify it and we are done.

$$\underbrace{x^2 + 6x + \left(\frac{6}{2}\right)^2}_{(x+\frac{6}{2})^2} + \underbrace{7 - \left(\frac{6}{2}\right)^2}_{7-9} = (x+3)^2 - 2$$

The result:

$$x^2 + 6x + 7 = (x+3)^2 - 2$$