Calc 3 Test 1 Review Buthusiem

1. Use the points P: (1, 2 ,3), Q: (-1, 0, 4), R:(2, -5, 6) to find $\overrightarrow{PQ}$, $\overrightarrow{PR}$, $\overrightarrow{QR}$ then find
   a. $\overrightarrow{PQ}\times\overrightarrow{PR}$ b. $\overrightarrow{PQ}\bullet\overrightarrow{PR}$ c. $\left(\overrightarrow{PQ}\times\overrightarrow{PR}\right)\bullet\overrightarrow{QR}$ and know what each can be used for.
2. Find the angle between the vector $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. How would you find the angle between planes?
3. Find the parametrization of the line through (1, 0, -3) and parallel to $\overrightarrow{PQ}$.
4. Find the equation of the plane (2, 0, 1) and perpendicular to the line $x=3t,\ y=2-t,\ z=3+4t$.
5. Identify the equation and give as many details describing the figure:
   a. $3x+4y-5z+43=0$ b. $x^2+y^2+z^2+9x-2y+10z+19=0$
6. Using $\vec{r}(t)=\left\langle e^{-t},\frac{1}{t},\frac{t}{t^2+1}\right\rangle$ Find the following:
   a. The domain of $\vec{r}(t)$ b. $\lim_{t\to\infty}\vec{r}(t)$ c. $\vec{r}'(t)$ d. $\int\vec{r}(t)dt$
7. If a particle has acceleration vector $\vec{a}(t)=-32k$ and the given initial conditions $\vec{v}(0)=\langle 3,-2,1\rangle$ and $\vec{r}(0)=\langle 0,5,2\rangle$. Find the velocity and position vectors.
8. Decompose the acceleration vector of $\vec{r}(t)=\left\langle t,\frac{1}{2}t^2,\frac{1}{6}t^3\right\rangle$ into the tangential and normal components when t = 0.
9. Identify the following surfaces:
   a. $y=x^2$ b. $x^2-y^2+z^2=1$ c. $z=y^2-x^2$
10. Find the arc length of $\vec{r}(t)=\langle t,3\cos t,3\sin t\rangle$, $-5\le t\le 5$.

(1.) $P:(1,2,3)$ $Q:(-1,0,4)$ $R:(2,-5,6)$

$\vec{PQ}=\langle -2,-2,1\rangle$ $\vec{PR}=\langle 1,-7,3\rangle$ $\vec{QR}:\langle 3,-5,2\rangle$

a.) $\vec{PQ}\times\vec{PR}=\begin{vmatrix} i & j & k \\ -2 & -2 & 1 \\ 1 & -7 & 3 \end{vmatrix}=(-6--7)i-(-6-1)j+(14--2)k$

$=\langle 1,7,16\rangle$

b.) $\vec{PQ}\cdot\vec{PR}=-2+14+3=15$

c.) $(\vec{PQ}\times\vec{PR})\cdot\vec{QR}=\langle 1,7,16\rangle\cdot\langle 3,-5,2\rangle$ or $\begin{vmatrix} 3 & -5 & 2 \\ -2 & -2 & 1 \\ 1 & -7 & 3 \end{vmatrix}=$

$=3+(-35)+32=0$

(2.) $\measuredangle$ btw $\vec{PQ}$ & $\vec{PR}$ use $\cos\theta=\dfrac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\|\vec{w}\|}\Rightarrow\cos\theta=\dfrac{15}{3\sqrt{59}}$

$\|\vec{PQ}\|=\sqrt{4+4+1}=3$

$\|\vec{PR}\|=\sqrt{1+49+9}=\sqrt{59}$

$\theta=\arccos\left(\dfrac{15}{3\sqrt{59}}\right)$

$\theta\approx 49.39^\circ$

(3.) line through $(1,0,-3)$ // to $\langle -2,-2,1\rangle$

use $\begin{aligned} x&=x_1+at \\ y&=y_1+bt \\ z&=z_1+ct \end{aligned}\Rightarrow\begin{aligned} x&=1-2t \\ y&=0-2t \\ z&=-3+t \end{aligned}$

(4.) Plane through $(2,0,1)$ $\perp$ to $x=3t$, $y=2-t$, $3+4t$

// vector to line is $\langle 3,-1,4\rangle$ this in normal to Plane

use $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

$3(x-2)-1(y-0)+4(z-1)=0$

or

$3x-y+4z-6+0-4=0$

$3x-y+4z-10=0$

(5) a. $3x+4y-5z+43=0$ is a plane w/ normal vector $\vec{n} = \langle 3, 4, -5 \rangle$

b. $x^2+y^2+z^2+9x-2y+10z+19=0$

$x^2+9x+(9/2)^2 + y^2-2y+(-1)^2 + z^2+10z+(5)^2 = -19+\frac{81}{4}+1+25$

$(x+9/2)^2+(y-1)^2+(z+5)^2 = \frac{109}{4}$

Sphere with center $(-9/2, 1, -5)$

& Radius $= \frac{\sqrt{109}}{2}$

6. $r(t) = \langle e^{-t}, \frac{1}{t}, \frac{t}{t^2+1} \rangle$

a. $D: \{t \mid t \neq 0\}$ or $(-\infty, 0) \cup (0, \infty)$

b. $\lim_{t\to\infty} r(t) = \langle \lim_{t\to\infty} e^{-t}, \lim_{t\to\infty} \frac{1}{t}, \lim_{t\to\infty} \frac{t}{t^2+1} \rangle$

$= \langle 0, 0, 0 \rangle$

c. $r'(t) = \langle \frac{d}{dt}[e^{-t}], \frac{d}{dt}[\frac{1}{t}], \frac{d}{dt}[\frac{t}{t^2+1}] \rangle$

$= \langle -e^{-t}, \frac{-1}{t^2}, \frac{(t^2+1)-2t(t)}{(t^2+1)^2} \rangle$

$= \langle -e^{-t}, \frac{-1}{t^2}, \frac{1-t^2}{(t^2+1)^2} \rangle$

d. $\int r(t)dt$

$= \int \langle e^{-t}, \frac{1}{t}, \frac{t}{t^2+1} \rangle dt = \langle e^{t}+C_1, \ln t + C_2, \frac{1}{2}\ln|t^2+1| + C_3 \rangle$

$\int e^{t}dt = e^{t}$ $\quad \int \frac{1}{t}dt = \ln t$ $\quad \int \frac{t}{t^2+1}dt = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u|$

$u = t^2+1$

$\frac{du}{2} = t\,dt$

7. if $a(t) = -32k$ then $v(t) = \int a(t)\,dt$

$\int a(t)\,dt = C_1 i + C_2 j + (-32t + C_3)k = v(t)$

if $v(0) = 3i - 2j + k$ then $3 = C_1$ $-2 = C_2$ $1 = -32(0) + C_3$
$1 = C_3$

So $\boxed{v(t) = 3i - 2j + (-32t + 1)k}$

$\int v(t)\,dt = r(t) = (3t + C_4)i + (-2t + C_5)j + (-16t^2 + t + C_6)k$

if $r(0) = 5j + 2k$ then $3(0) + C_4 = 0$ $-2(0) + C_5 = 5$ $-16(0)^2 + (0) + C_6 = 2$
$C_4 = 0$ $C_5 = 5$ $C_6 = 2$

so $\boxed{r(t) = 3ti + (-2t + 5)j + (-16t^2 + t + 2)k}$

8. $r(t) = \langle t, \frac{1}{2}t^2, \frac{1}{6}t^3 \rangle$ $t = 1$

$r'(t) = v(t) = \langle 1, t, \frac{1}{2}t^2 \rangle$ $r'(1) = \langle 1, 1, \frac{1}{2} \rangle = v(1)$

$r''(t) = a(t) = \langle 0, 1, t \rangle$ $r''(1) = \langle 0, 1, 1 \rangle = a(1)$

$T(t) = \frac{r'(t)}{\|r'(t)\|} = \frac{\langle 1, t, \frac{1}{2}t^2 \rangle}{\sqrt{1 + t^2 + \frac{1}{4}t^4}}$ $T(1) = \frac{\langle 1, 1, \frac{1}{2} \rangle}{\sqrt{1 + 1 + \frac{1}{4}}} = \langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle$

$a_T = a \cdot T$ at $t = 1$ $a_T = \langle 0, 1, 1 \rangle \cdot \langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle$
$= 0 + \frac{2}{3} + \frac{1}{3} = 1$

$a = a_T T + a_N N$

$a_N N = a - a_T T$ at $t = 1$ $a_N N = \langle 0, 1, 1 \rangle - 1\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle$
$= \langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3} \rangle$

$a_N = \|a_N N\|$
at $t = 1$ $a_N = \sqrt{\frac{4}{9} + \frac{1}{9} + \frac{4}{9}} = 1$

$N = \frac{a_N N}{a_N} = \frac{\langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3} \rangle}{1} = \langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3} \rangle$

$\therefore a = a_T T + a_N N$ at $t = 1$ $a = 1\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle + 1\langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$

(9.) see video for solution

(10.) arc length $\vec{r}(t) = \langle t, 3\cos t, 3\sin t \rangle$, $-5 \le t \le 5$

$$S = \int_{-5}^{5} \|r'(t)\| dt$$

$$r'(t) = \langle 1, -3\sin t, 3\cos t \rangle$$

$$\|r'(t)\| = \sqrt{1 + 9\sin^2 t + 9\cos^2 t} = \sqrt{1 + 9(\sin^2 t + \cos^2 t)}$$

$$= \sqrt{1+9}$$

$$= \sqrt{10}$$

$$S = \int_{5}^{5} \sqrt{10}\, dt = \sqrt{10}\, t \Big|_{-5}^{5}$$

$$= \sqrt{10}(5 - -5) = 10\sqrt{10}$$