Calc II Test #1 Problem Set Bathusiem

1. Find the Following functions derivatives.

a. $y = \ln\sqrt{\dfrac{x^2+4}{x^2-4}}$ b. $y = \sqrt{e^{2x}+e^{-2x}}$

c. $y = x^{2x+1}$ d. $y = \tan(\arcsin x)$

2. Find the following integrals.

a. $\displaystyle\int \frac{\ln\sqrt{x}}{x}\,dx$ b. $\displaystyle\int \frac{e^{4x}-e^{2x}+1}{e^{x}}\,dx$

c. $\displaystyle\int \frac{1}{e^{2x}+e^{-2x}}\,dx$ d. $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\,dx$

e. $\displaystyle\int \frac{\arctan(x/2)}{4+x^2}\,dx$ f. $\displaystyle\int \frac{dx}{x^2+4x+13}$

g. $\displaystyle\int \frac{x^3-6x-20}{x+5}\,dx$

3. Find the area bounded by $f(x)=\sqrt{x}+3$ and $g(x)=\frac{1}{2}x+3$

4. Find the Volume of the Solid generated by revolving the region bounded by $y=2x^2$, $y=0$, and $x=2$ about the given lines

a. the y-axis b. the x-axis

-. the line $y=8$

5. Find the volume of the solid generated by revolving the region bounded by $y = 5x - x^2$ & $y = 0$ about the line $x = -1$

6. Find the arc length of the graph of the function $y = 2x^{3/2} + 3$ over $[0, 9]$

Calc II Test #1 Problem Set KEY G. Buthusiem

1.a. $y = \ln\sqrt{\dfrac{x^2+4}{x^2-4}}$

$y = \frac{1}{2}\left(\ln(x^2+4) - \ln(x^2-4)\right)$

$y' = \frac{1}{2}\left(\dfrac{2x}{x^2+4} - \dfrac{2x}{x^2-4}\right)$

$y' = \dfrac{x}{x^2+4} - \dfrac{x}{x^2-4}$

1b. $y = \sqrt{e^{2x}+e^{-2x}}$

$y' = \frac{1}{2}\left(e^{2x}+e^{-2x}\right)^{-1/2}\left(2e^{2x}-2e^{-2x}\right)$

$y' = \dfrac{e^{2x}-e^{-2x}}{\left(e^{2x}+e^{-2x}\right)^{1/2}}$

1c. $y = x^{2x+1}$

$\ln y = \ln x^{2x+1}$

$\ln y = (2x+1)\ln x$

$\dfrac{y'}{y} = 2\ln x + \dfrac{2x+1}{x}$

$y' = x^{2x+1}\left(\dfrac{2x\ln x + 2x+1}{x}\right)$

$y' = x^{2x}(2x\ln x + 2x + 1)$

1d. $y = \tan(\arcsin x)$

$y' = \sec^2(\arcsin x)\cdot\dfrac{1}{\sqrt{1-x^2}}$

2. a. $\displaystyle\int \frac{\ln\sqrt{x}}{x}\,dx$ let $u = \ln\sqrt{x}$

$du = \dfrac{\frac{1}{2}x^{-1/2}}{x^{1/2}}\,dx$

$2du = \dfrac{1}{\cancel{2}}\cdot\dfrac{1}{x}\,dx$

$\rightarrow \displaystyle\int 2u\,du = u^2 + C$

$= (\ln\sqrt{x})^2 + C$

b. $\displaystyle\int \frac{e^{4x}-e^{2x}+1}{e^x}\,dx = \int\left(e^{3x}-e^x+e^{-x}\right)dx = \frac{e^{3x}}{3} - e^x - e^{-x} + C$

c. $\displaystyle\int \frac{1}{e^{2x}+e^{-2x}}\,dx = \int \frac{1}{e^{2x}+\frac{1}{e^{2x}}}\,dx = \int \frac{1}{\frac{e^{4x}+1}{e^{2x}}}\,dx = \int \frac{e^{2x}}{e^{4x}+1}\,dx$

let $u = e^{2x}$

$\dfrac{du}{2} = \cancel{2}e^{2x}\,dx$

$\rightarrow \displaystyle\frac{1}{2}\int \frac{du}{u^2+1} = \frac{1}{2}\arctan(e^{2x}) + C$

2d. $\int \frac{x}{\sqrt{1-x^4}}\,dx$ let $u = x^2$

$du = 2x\,dx$

$\frac{du}{2}$

$\Rightarrow = \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\arcsin(x^2) + C$

2e. $\int \frac{\arctan(x/2)}{4+x^2}\,dx$ let $u = \arctan(x/2)$

$du = \frac{1/2}{1+(\frac{x}{2})^2}\,dx = \frac{2}{4+x^2}\,dx$

$= \frac{1}{2}\int u\,du = \frac{1}{2}\frac{u^2}{2} + C = \frac{\arctan^2(x/2)}{4} + C$

2f. $\int \frac{dx}{x^2+4x+13}$

USE

$x^2+4x \quad +13$

$= x^2+4x+(2)^2+13-(2)^2$

$= (x+2)^2+9$

$= \int \frac{dx}{(x+2)^2+9}$ let $u = x+2$ $a=3$

$du = dx$

$\Rightarrow \int \frac{du}{u^2+a^2}$

$= \frac{1}{3}\arctan\left(\frac{x+2}{3}\right) + C$

2g. $\int \frac{x^3-6x-20}{x+5}\,dx$

use: $x+5 \overline{)\,x^3+0x^2-6x-20}$ with quotient $x^2-5x+19$

$-(x^3+5x^2)$

$-5x^2-6x$

$-(-5x^2-25x)$

$19x-20$

$-(19x+95)$

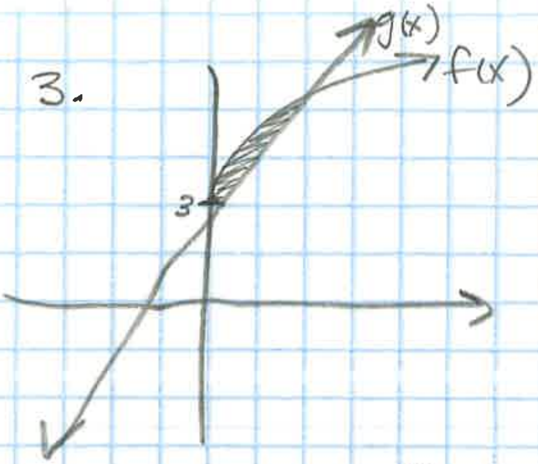
-115

$= \int\left(x^2-5x+19 - \frac{115}{x+5}\right)dx$

$= \frac{x^3}{3} - \frac{5x^2}{2} + 19x - 115\ln|x+5| + C$

(Hint For test: how would you do $\int \frac{3x+7}{\sqrt{4-x^2}}\,dx$)

3.



Int Points

$\sqrt{x}+3=\frac{1}{2}x+3$

$\sqrt{x}=\frac{1}{2}x$

$x=\frac{1}{4}x^2$

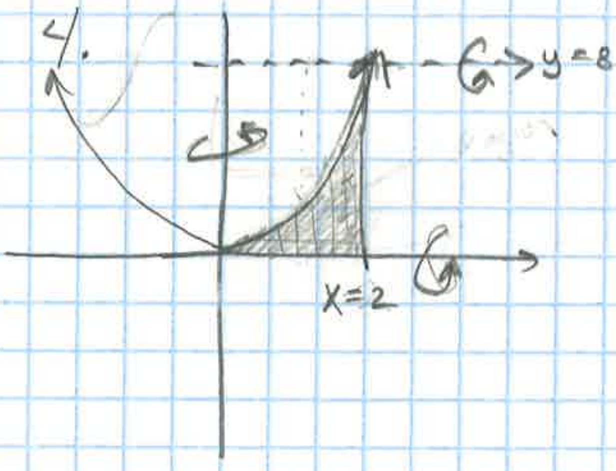
$0=x(\frac{1}{4}x-1)$

$x=0 \quad x=4$

$$A=\int_0^4(\sqrt{x}+3-(\tfrac{1}{2}x+3))dx=\int_0^4(\sqrt{x}-\tfrac{1}{2}x)dx$$

$$=\frac{2}{3}x^{3/2}-\frac{x^2}{4}\Big|_0^4=\frac{2}{3}\cdot4^{3/2}-\frac{4^2}{4}=\frac{16}{3}-\frac{16}{4}=\boxed{\frac{4}{3}}$$

4.



a. about the y-axis use Shell Method

$$V=2\pi\int_0^2 x(2x^2)dx=2\pi\int_0^2 2x^3dx$$

$$=2\pi\left(\frac{x^4}{2}\right)\Big|_0^2=2\pi\left(\frac{16}{2}\right)=16\pi$$

b. about the x-axis use Disk Method

$$V=\pi\int_0^2(2x^2)^2dx=\pi\int_0^2 4x^4dx=\pi\frac{4x^5}{5}\Big|_0^2=\pi\left(\frac{4(32)}{5}\right)$$

$$=\frac{128}{5}\pi$$

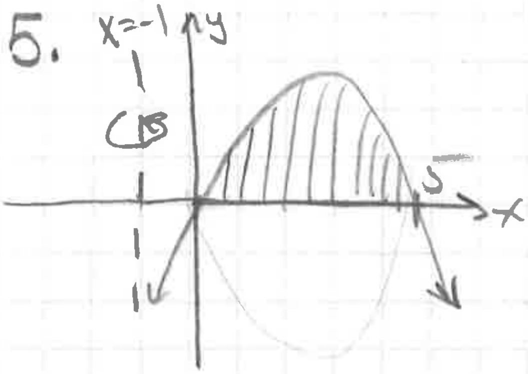
c. about $y=8$ use Washer. Method

$$V=\pi\int_0^2(8^2-(8-2x^2)^2)dx=\pi\int_0^2(64-64+32x^2-4x^4)dx$$

$$=\pi\int_0^2(32x^2-4x^4)dx=\pi\left(\frac{32x^3}{3}-\frac{4x^5}{5}\right)\Big|_0^2=\pi\left(\frac{32(8)}{3}-\frac{4(32)}{5}\right)$$

$$=\frac{896}{15}\pi$$

5.



Use Shell Method

$$V = 2\pi \int_0^5 (x+1)(5x - x^2)\,dx$$

$$= 2\pi \int_0^5 (-x^3 + 4x^2 + 5x)\,dx$$

$$= 2\pi \left(\frac{-x^4}{4} + \frac{4x^3}{3} + \frac{5x^2}{2} \right) \Big|_0^5$$

$$= 2\pi \left(\frac{-5^4}{4} + \frac{4(5)^3}{3} + \frac{5(5)^2}{2} \right)$$

$$= 2\pi \left(\frac{875}{12} \right) = \frac{875\pi}{6}$$

6.

$$\frac{5}{4} + \frac{4}{3} + \frac{1}{2}$$

$$-15 + 16 + 6$$