Calc 2 Final exam Review G. Buthusiem

1. Evaluate the following integrals

a. $\int_1^e \frac{\ln x}{x}\,dx$ b. $\int \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$ c. $\int \frac{1}{x^2-4x+7}\,dx$

d. $\int \cos^3\theta \sin^8\theta\,d\theta$ e. $\int x^2 e^{-12x}\,dx$ f. $\int \frac{dx}{x(x^2-1)^{3/2}}$

g. $\int \frac{4x+4}{(x-5)(x+3)}\,dx$ h. $\int_2^5 (5-x)^{-1/3}\,dx$

2. Find V obtained by rotating the region bounded by $f(x)=2x-x^2$ and the x axis, about the y-axis.

3. Find the interval of convergence. for $\sum_{n=0}^{\infty} \frac{8^n}{n!}x^n$

4. Find the power series expansion for $f(x)=\frac{1}{2+x^2}$

5. Find the Taylor series expansion for $f(x)=\ln(1-x^2)$ centered at zero & find the interval of convergence.

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1. a. $\int_1^e \frac{\ln x}{x}\,dx = \frac{(\ln x)^2}{2}\Big|_1^e = \frac{(\ln e)^2}{2} - \frac{(\ln 1)^2}{2} = \frac{1}{2}$

$u = \ln x \quad du = \frac{1}{x}\,dx$

b. $\int \frac{\arcsin x}{\sqrt{1-x^2}}\,dx = \frac{(\arcsin x)^2}{2} + C$

$u = \arcsin x \quad du = \frac{1}{\sqrt{1-x^2}}\,dx$

c. $\int \frac{1}{x^2-4x+7}\,dx = \int \frac{1}{(x-2)^2+3}\,dx = \frac{1}{\sqrt{3}}\arctan\left(\frac{x-2}{\sqrt{3}}\right) + C$

$x^2-4x+7 = x^2-4x+(-2)^2+7-(-2)^2$
$= (x-2)^2+3$

d. $\int \cos^3\theta \sin^8\theta\,d\theta = \int \cos^2\theta \sin^8\theta \cos\theta\,d\theta$

$= \int (1-\sin^2\theta)\sin^8\theta\cos\theta\,d\theta = \int (\sin^8\theta - \sin^{10}\theta)\cos\theta\,d\theta$

$= \frac{\sin^9\theta}{9} - \frac{\sin^{11}\theta}{11} + C$

e. $\int x^2 e^{-12x}\,dx = -\frac{x^2 e^{-12x}}{12} - \frac{2x e^{-12x}}{144} - \frac{e^{-12x}}{864} + C$

$x^2 \quad + \quad e^{-12x}$

$2x \quad - \quad \frac{e^{-12x}}{-12}$

$2 \quad + \quad \frac{e^{-12x}}{144}$

$\frac{e^{-12x}}{(-12)^3}$

f. $\int \frac{dx}{x(x^2-1)^{3/2}} \longrightarrow \int \frac{\sec\theta\tan\theta\,d\theta}{\sec\theta\tan^3\theta} = \int \frac{1}{\tan^2\theta}\,d\theta$

use $\sqrt{x^2-1} = \tan\theta$
$x = \sec\theta$
$dx = \sec\theta\tan\theta\,d\theta$

$= \int \cot^2\theta\,d\theta = \int (\csc^2\theta - 1)\,d\theta$

$= -\cot\theta - \theta + C$

$\longrightarrow$ see next Page

$$= -\frac{1}{\sqrt{x^2-1}} - \operatorname{arcsec} x + C$$

g. $\displaystyle\int \frac{4x+4}{(x-5)(x+3)}\,dx \longrightarrow = \int\left(\frac{3}{x-5} + \frac{1}{x+3}\right)dx$

$$\frac{4x+4}{(x-5)(x+3)} = \frac{A}{x-5} + \frac{B}{x+3}$$

$$= 3\ln|x-5| + \ln|x+3| + C$$

$$= \ln\left|(x-5)^3\cdot(x+3)\right| + C$$

$$4x+4 = A(x+3) + B(x-5)$$

let $x = -3$ let $x = 5$

$-8 = B(-8)$ $24 = A(8)$

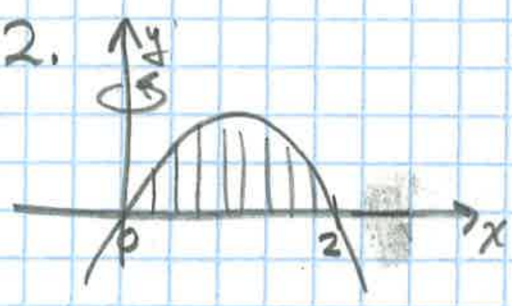
$B = 1$ $A = 3$

h. $\displaystyle\int_2^5 (5-x)^{-1/3}\,dx = \lim_{b\to5^-}\int_2^b (5-x)^{-1/3}\,dx$

$$= \lim_{b\to5^-}\left(-\frac{3}{2}(5-x)^{2/3}\Big|_2^b\right) = \lim_{b\to5^-}\left(-\frac{3}{2}\left((5-b)^{2/3} - (3)^{2/3}\right)\right)$$

$$= \frac{3}{2}(3)^{2/3} = \frac{3^{5/3}}{2}$$

2.



Shell Method

$$V = 2\pi\int_a^b \text{radius}\cdot\text{height}\,dx$$

$$= 2\pi\int_0^2 x(2x-x^2)\,dx$$

$$= 2\pi\int_0^2 (2x^2-x^3)\,dx = 2\pi\left(\frac{2x^3}{3} - \frac{x^4}{4}\right)\Big|_0^2$$

$$= 2\pi\left(\frac{2(8)}{3} - \frac{16}{4}\right) = 2\pi\left(\frac{4}{3}\right) = \frac{8\pi}{3}$$

3. $\sum_{n=0} \frac{8^n}{n!} x^n$

$\lim_{n\to\infty} \left| \frac{8^{n+1}}{(n+1)!} x^{n+1} \cdot \frac{n!}{8^n x^n} \right| = \lim_{n\to\infty} \left| \frac{8x}{n+1} \right| = |8x| \lim_{n\to\infty} \left| \frac{1}{n+1} \right|$

$= 8|x| \cdot 0$ when <1 converges by Ratio Test.

$\therefore$ Interval of Convergence is $(-\infty, \infty)$

4. $f(x) = \frac{1}{2+x^2} = \frac{1}{2(1+\frac{x^2}{2})} = \frac{1/2}{1-(-\frac{x^2}{2})}$ let $a = 1/2$, $r = -\frac{x^2}{2}$

then $f(x) = \frac{1}{2+x^2} = \sum_{n=0}^{\infty} \frac{1}{2} \cdot \left(-\frac{x^2}{2}\right)^n = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{n+1}}$

5. $f(x) = \ln(1-x^2) \longrightarrow f(0) = \ln 1 = 0$

$f'(x) = \frac{-2x}{1-x^2}$ $\quad f'(0) = 0$

$f''(0) = -2$

$f''(x) = \frac{-2(1-x^2)+2x(-2x)}{(1-x^2)^2} = \frac{-2x^2-2}{(1-x^2)^2}$ $\quad f'''(0) = 0$

$f'''(x) = \frac{-4x(1-x^2)^2 + (2x^2+2)(2(1-x^2)(-2x))}{(1-x^2)^2}$

$= \frac{-4x(1-x^2) + (2x^2+2)(-4x)}{(1-x^2)^2} = \frac{-4x(x^2+3)}{(1-x^2)^3}$

$f^{(4)}(x) = \frac{-12(x^4+6x^2+1)}{(x^2-1)^4}$ $\quad f^{(4)}(0) = -12$

$f^{(5)}(x) = \frac{48x(x^4+10x^2+5)}{(x^2-1)^5}$ $\quad f^{(5)}(0) = 0$

$f^{(6)}(x) = \frac{-240(x^6+15x^4+15x^2+1)}{(x^2-1)^6}$ $\quad f^{(6)}(0) = -240$

$\therefore f(x) = \ln(1-x^2) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^2 =$ $\rightarrow$ see next Page

$$= f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^4 + \frac{f^{(4)}(0)}{4!}x^4 + \cdots$$

$$= 0 + 0 - \frac{2}{2!}x^2 + 0 - \frac{12}{4!}x^4 + 0 - \frac{240}{6!}x^6 + \cdots$$

$$= -x^2 - \frac{1}{2}x^4 - \frac{1}{3}x^6 - \frac{1}{4}x^8 \cdots$$

$$= \sum_{n=1}^{\infty} \frac{(-1)x^{2n}}{n}$$

$$\lim_{n\to\infty}\left|\frac{(-1)x^{2(n+1)}}{n+1} \cdot \frac{n}{(-1)x^{2n}}\right| = \lim_{n\to\infty}\left|x^2 \cdot \frac{n}{n+1}\right| = |x^2| < 1$$

Converge by Ratio Test.

$-1 < x < 1$ Check $x = -1$

$\sum_{n=1}^{\infty} \frac{(-1)}{n}$ converges by Alternating Series test

$x = 1$ $\sum_{n=1}^{\infty} \frac{(-1)}{n}$ converges by Alt. Series Test

$\therefore$ Int of Convergence is $[-1, 1]$