Calc I Test #3 Problem Set G. Buthusiem.

1. Evaluate $\int_0^1 3x\,dx$ using $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

2. Evaluate the following indefinite integrals:

a. $\int \frac{x^2+2x-6}{x^4}\,dx$ b. $\int (5\cos x - 2\sec^2 x)\,dx$

c. $\int 6x^3\sqrt{3x^4+2}\,dx$ d. $\int \frac{x+4}{x^2+8x-7}\,dx$ e. $\int \sin^4 5x \cos 5x\,dx$

3. Evaluate the following definite integrals:

a. $\int_0^3 (5x^3+3x-1)\,dx$ b. $\int_0^1 (3x+1)^5\,dx$ c. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin 2x\,dx$

4. Find the area bounded by $f(x) = -x^2+x+6$, the x-axis, the lines $x = -2$ and $x = 3$

5. Find the following derivatives:

a. $y = x^2 \ln x$ b. $y = \ln\sqrt[3]{\frac{x-1}{x+1}}$ c. $y = 5e^{x^2+5}$

6. Use log differentiation to find y' for $y = \frac{(x+2)^2}{3x(x-1)^3}$

7. Find the following indefinite integrals:

a. $\int e^x(e^x+1)^2\,dx$ b. $\int \frac{5-e^x}{e^{2x}}\,dx$ c. $\int \frac{x^2+2x+3}{x^3+3x^2+9x}\,dx$

d. $\int \frac{\cos t}{1+\sin t}\,dt$

Calc I test #3 Problem Set KEY G. Buthusiem

1. $\int_0^1 3x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \rightarrow f(x) = 3x \quad [0,1]$

$\Delta x = \frac{1}{n} \quad x_i = a + i\Delta x = \frac{i}{n} \quad f(x_i) = f(i/n) = 3i/n$

$\sum f(x_i)\Delta x = \sum_{i=1}^{n} \frac{3i}{n}(1/n) = \frac{3}{n^2}\sum_{i=1}^{n} i = \frac{3}{n^2}\left(\frac{n(n+1)}{2}\right)$

$\lim_{n\to\infty} \sum f(x_i)\Delta x = \lim_{n\to\infty} \frac{3}{n^2}\frac{(n)(n+1)}{2} = 3/2$

2. a. $\int \frac{x^2+2x-6}{x^4}\,dx = \int\left(\frac{x^2}{x^4} + \frac{2x}{x^4} - \frac{6}{x^4}\right)dx = \int\left(x^{-2} + 2x^{-3} - 6x^{-4}\right)dx$

$= \frac{x^{-1}}{-1} + \frac{2x^{-2}}{-2} - \frac{6x^{-3}}{-3} + C = -\frac{1}{x} - \frac{1}{x^2} + \frac{2}{x^3} + C$

b. $\int (5\cos x - 2\sec^2 x)\,dx = 5\int \cos x\,dx - 2\int \sec^2 x\,dx$

$= 5\sin x - 2\tan x + C$

c. $\int 6x^3\sqrt{3x^4+2}\,dx = \frac{6}{12}\int u^{1/2}\,du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$

$u = 3x^4+2 \quad \frac{du}{12} = 12x^3\,dx$

$= \frac{1}{3}(3x^4+2)^{3/2} + C$

d. $\int \frac{x+4}{x^2+8x-7}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+8x-7| + C$

$u = x^2+8x-7$

$\frac{du}{2} = (2x+8)\,dx = \cancel{2}(x+4)\,dx$

e. $\int \sin^4 5x\cos 5x\,dx = \frac{1}{5}\int u^4\,du = \frac{1}{5}\frac{u^5}{5} + C$

$u = \sin 5x$

$\frac{du}{5} = \cancel{5}\cos 5x\,dx$

$= \frac{1}{25}\sin^5 5x + C$

3. a. $\int_0^3 (5x^3+3x-1)\,dx = \frac{5x^4}{4} + \frac{3x^2}{2} - x \Big|_0^3 = \frac{5}{4}(3)^4 + \frac{3(3)^2}{2} - 3$

$= 3\left(\frac{5\cdot 27}{4} + \frac{9}{2} - 1\right) = 3\left(\frac{135+18-4}{4}\right) = \frac{447}{4}$

b. $\int_0^1 (3x+1)^5\,dx = \frac{1}{3}\int_0^1 u^5\,du = \frac{1}{18}u^6 \Big|_0^1 = \frac{1}{18}(3x+1)^6 \Big|_0^1$

$u = 3x+1$

$du = 3dx$

$\frac{du}{3}$

$= \frac{1}{18}\left((3+1)^6 - (0+1)^6\right) = \frac{4095}{18}$

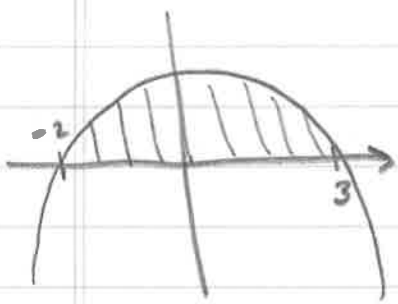
c. $\int_{-\pi/4}^{\pi/4} \sin 2x\,dx = \frac{1}{2}\int_{-\pi/4}^{\pi/4} \sin u\,du = -\frac{1}{2}\cos 2x \Big|_{-\pi/4}^{\pi/4}$

$u = 2x$ $\quad du = 2dx$ $\quad \frac{du}{2}$

$= -\frac{1}{2}\left(\cos(2(\pi/4)) - \cos(2(-\pi/4))\right)$

$= -\frac{1}{2}(0 - 0) = 0$

4. i. $f(x) = -x^2 + x + 6 \quad [-2, 3] \rightarrow$ f(x) has x int. at x=-2 & x=3 and is a downward parabola



$A = \int_{-2}^{3} (-x^2+x+6)\,dx = -\frac{x^3}{3} + \frac{x^2}{2} + 6x \Big|_{-2}^{3}$

$= -\frac{3^3}{3} + \frac{3^2}{2} + 6(3) - \left(-\frac{(-2)^3}{3} + \frac{(-2)^2}{2} + 6(-2)\right)$

$= -\frac{27}{3} + \frac{9}{2} + 18 - \frac{8}{3} - 4/2 + 12 = 19 + \frac{9}{2} - \frac{8}{3} = \boxed{\frac{125}{6}}$

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5. a. $y = x^2 \ln x \rightarrow y' = 2x\ln x + x^2 \cdot \frac{1}{x} = 2x\ln x + x$

b. $y = \ln \sqrt[3]{\frac{x-1}{x+1}} = \frac{1}{3}(\ln(x-1) - \ln(x+1))$

$y' = \frac{1}{3}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$

c. $y = 5e^{x^2+5} \longrightarrow y' = 5e^{x^2+5}(2x) = 10xe^{x^2+5}$

6. $y = \frac{(x+2)^2}{3x(x-1)^3}$

$\ln y = \ln\left(\frac{(x+2)^2}{3x(x-1)^3}\right) = 2\ln(x+2) - \ln 3x - 3\ln(x-1)$

$y'/y = 2 \cdot \frac{1}{x+2} - \frac{3}{3x} - \frac{3}{x-1}$

$y' = y\left(\frac{2}{x+2} - \frac{1}{x} - \frac{3}{x-1}\right) = \frac{(x+2)^2}{3x(x-1)^3}\left(\frac{2x(x-1) - (x+2)(x-1) - 3x(x+2)}{x(x+2)(x-1)}\right)$

$y' = \frac{(x+2)(-2x^2-9x+2)}{3x^2(x-1)^4}$

7a. $\int e^x(e^x+1)^2\,dx = \int u^2\,du = \frac{(e^x+1)^3}{3} + C$

$u = e^x + 1$

$du = e^x\,dx$

b. $\int \frac{5-e^x}{e^{2x}}\,dx = \int\left(\frac{5}{e^{2x}} - \frac{e^x}{e^{2x}}\right)dx = \int(5e^{-2x} - e^{-x})\,dx$

$= 5\int e^{-2x}\,dx - \int e^{-x}\,dx = -\frac{5}{2}e^{-2x} + e^{-x} + C$

$u = -2x \quad du = -2dx \qquad u = -x \quad du = -dx$

c. $\int \frac{x^2+2x+3}{x^3+3x^2+9x}\,dx = = \frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln|x^3+3x^2+9x| + C$

$u = x^3 + 3x^2 + 9x$

$du = (3x^2+6x+9)\,dx$

$\frac{du}{3} = \cancel{3}(x^2+2x+3)\,dx$

7.d. $\int \frac{\cos t}{1+\sin t}\,dt = \int \frac{du}{u} = \ln|1+\sin t| + C$

$u = 1+\sin t$

$du = \cos t\,dt$