Calc 1 Test #2 Review G.Buthusiem

1. Find any critical numbers of the function $g(t) = t\sqrt{10-t}$, $t < 10$.
2. Find all critical numbers of the function $f(x) = \sin^2 7x + \cos 7x$, $0 < x < \frac{2\pi}{7}$.
3. Locate the absolute extrema of the function $f(x) = 3x^2 - 12x + 4$ on the closed interval $[-4, 4]$.
4. Determine whether Rolle's Theorem can be applied to $f(x) = -x^2 + 12x$ on the closed interval $[0,12]$. If Rolle's Theorem can be applied, find all values of c in the open interval $(0,12)$ such that $f'(c) = 0$.
5. Determine whether the Mean Value Theorem can be applied to the function $f(x) = x^2$ on the closed interval [7,15]. If the Mean Value Theorem can be applied, find all numbers c in the open interval (7,15) such that $f'(c) = \frac{f(15) - f(7)}{15 - 7}$.
6. Identify the open intervals on which the function $y = 12x - 24\cos x,\ 0 < x < 2\pi$ is increasing or decreasing.
7. Find the relative extremum of $f(x) = -7x^2 + 42x + 4$ by applying the First Derivative Test.
8. Determine the open intervals on which the graph of $y = -7x^3 + 7x^2 + 2x - 3$ is concave downward or concave upward.
9. Determine the open intervals on which the graph of the function $f(x) = \frac{x^2}{x^2 + 16}$ is concave upward or concave downward.
10. Find the points of inflection and discuss the concavity of the function $f(x) = x\sqrt{x+4}$.
11. Find the limit.
    $\lim_{x \to \infty}\left(-4 + \frac{1}{x^6}\right)$
12. Find the limit.
    $\lim_{x \to \infty}\frac{-2x + 4}{7x^2 + 7}$
13. Find the length and width of a rectangle that has perimeter 40 meters and a maximum area.
14. Determine the dimensions of a rectangular solid (with a square base) with maximum volume if its surface area is 400 meters.

Answer Key

1. $\frac{20}{3}$
2. $\frac{\pi}{21}, \frac{\pi}{7}, \frac{5\pi}{21}$
3. absolute max: (–4, 100) ; absolute min: (2, –8)
4. $c = 6$
5. MVT applies; $c = 11$
6. increasing on $\left(0, \frac{7\pi}{6}\right)$ and $\left(\frac{11\pi}{6}, 2\pi\right)$; decreasing on $\left(\frac{7\pi}{6}, \frac{11\pi}{6}\right)$
7. relative maximum: $(3, 67)$
8. concave upward on $\left(-\infty, \frac{1}{3}\right)$; concave downward on $\left(\frac{1}{3}, \infty\right)$
9. concave upward: $\left(-\frac{4\sqrt{3}}{3}, \frac{4\sqrt{3}}{3}\right)$; concave downward: $\left(-\infty, -\frac{4\sqrt{3}}{3}\right)$, $\left(\frac{4\sqrt{3}}{3}, \infty\right)$
10. no inflection points; concave up on $(-4, \infty)$
11. –4
12. 0
13. 10, 10
14. square base side $\frac{10\sqrt{6}}{3}$; height $\frac{10\sqrt{6}}{3}$