Calc I Test #2 Problem Set.

1. Find the absolute extrema of $f(x)=3\sqrt{x}-x$ on $[0,16]$

2. Find the critical #'s of $f'(x)$ for $f(x)=\dfrac{x}{\sqrt{x^2+1}}$

3. Find the open intervals for wich the following functions are increasing & decreasing as well as the relative extrema if they exist.

   a. $f(x)=(x-1)^2(x-3)$ b. $y=\sin x+\cos x$ on $(0,2\pi)$

4. Use the Second derivative test to find all relative extrema if possible, for $f(x)=x^3-x$

5. Determine the concauity & find the points of inflection for the following.

   a. $f(x)=x^3-12x-16$ b. $f(x)=\dfrac{2x}{1+x^2}$

6. Use the Mean Value Theorem to find all values of for wich the average slope is equal to the instanteous slope for the following function on the interval for $f(x)=x^4-8x$, over $[0,2]$

7. Analyze the graph of $y=\dfrac{5-3x}{x-2}$

8. Find the limit

   a. $\displaystyle\lim_{x\to\infty}\frac{5-3x}{x-2}$ b. $\displaystyle\lim_{x\to-\infty}\frac{3x^2+1}{1-x}$ c. $\displaystyle\lim_{x\to-\infty}\frac{-3x}{\sqrt{x^2+4}}$

9. A cylindrical can is made to hold 1L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

   Hint: ($1L = 1000\,cm^3$)

Calc I Test #2 Problem Set Key G. Buthusiem

1. $f(x) = 3x^{1/2} - x \quad [0, 16]$

$f'(x) = \frac{3}{2}x^{-1/2} - 1 = \frac{3}{2x^{1/2}} - 1 = \frac{3 - 2x^{1/2}}{2x^{1/2}}$

Critical #'s

$3 - 2x^{1/2} = 0 \qquad 2x^{1/2} = 0$

$x = 9/4 \qquad x = 0$

$\rightarrow$ Find $f(0) = 0$

$f(9/4) = 3(9/4)^{1/2} - 9/4 = 9/4$ Biggest

& $f(16) = 3(16)^{1/2} - 16 = -4$ smallest.

$\therefore$ f(x) has absolute max at $(9/4, 9/4)$

& f(x) has an absolute min at $(16, -4)$

2. $f(x) = \frac{x}{\sqrt{x^2+1}}$ $\qquad f'(x) = \frac{1(x^2+1)^{1/2} - x\left(\frac{1}{2}(x^2+1)^{-1/2}(2x)\right)}{x^2+1}$

$= \frac{(x^2+1)^{1/2} - \frac{x^2}{(x^2+1)^{1/2}}}{x^2+1} = \frac{x^2+1-x^2}{(x^2+1)^{3/2}}$

$= \frac{1}{(x^2+1)^{3/2}}$

Critical #'s: $f'(x) = 0$ or $f'(x)$ DNE

$f'(x)$ is never zero & $f'(x)$ always exists for $x \geq 0$

$\therefore$ there are no critical #'s.

3. a. $f(x) = (x-1)^2(x-3)$ you could multiply this out if you want.

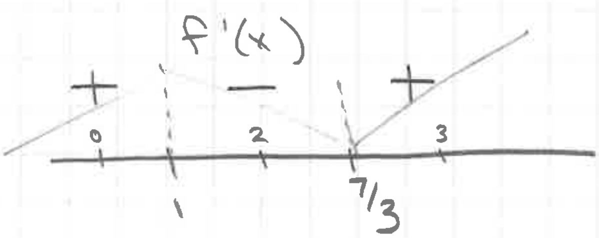
$f'(x) = 2(x-1)^1(x-3) + (x-1)^2(1)$ $\leftarrow$ Factor out an $(x-1)$

$= (x-1)(2(x-3) + (x-1)) = (x-1)(3x-7)$

Critical #'s $f'(x) = 0 \quad f'(x)$ DNE

$(x-1)(3x-7) = 0$

$x = 1 \quad x = 7/3$



$\therefore$ f(x) is inc $(-\infty, 1)$ & $(7/3, \infty)$

f(x) is dec $(1, 7/3)$

& f(x) has a max at $(1, f(1))$

f(x) has a min at $(7/3, f(7/3))$

3b. $y = \sin x + \cos x$

$y' = \cos x - \sin x$ $\longrightarrow$

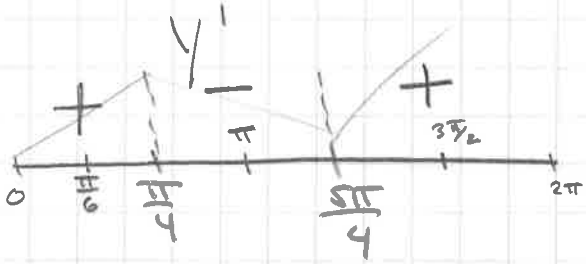
Critical #'s

$\cos x - \sin x = 0$

$\cos x = \sin x$

$1 = \dfrac{\sin x}{\cos x} = \tan x$

$x = \pi/4 \,,\ \dfrac{5\pi}{4}$



$\therefore f(x)$ is INC on $(0, \pi/4)$ & $(\frac{5\pi}{4}, 2\pi)$

$f(x)$ is DEC on $(\pi/4, 5\pi/4)$

$f(x)$ has a max at $(\pi/4, f(\pi/4))$

$f(x)$ has a min at $(5\pi/4, f(5\pi/4))$

4. $f(x) = x^3 - x$ $\quad f'(x) = 3x^2 - 1$

Critical #'s

$3x^2 - 1 = 0 \quad x = \pm\sqrt{1/3}$

possible relalite extrema

$f''(x) = 6x \quad f''(\sqrt{1/3}) = 6(\sqrt{1/3}) = +$

$f''(-\sqrt{1/3}) = 6(-\sqrt{1/3}) = -$

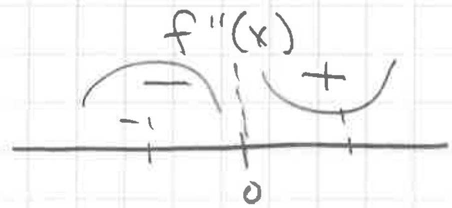
Since $f''(x)$ is $+$ at $x = \sqrt{1/3}$ $f(x)$ has a min at $(\sqrt{1/3}, f(\sqrt{1/3}))$ and since $f''(x)$ is $-$ at $x = -\sqrt{1/3}$ $f(x)$ has a max at $(-\sqrt{1/3}, f(-\sqrt{1/3}))$

5. a. $f(x) = x^3 - 12x - 16 \quad f'(x) = 3x^2 - 12 \quad f''(x) = 6x$

Critical #'s for $f''(x)$

$6x = 0$

$x = 0$



$\therefore f(x)$ is concave down $(-\infty, 0)$ & concave up $(0, \infty)$

$f(x)$ has a point of inflection at $(0, f(0))$

5.b $f(x) = \dfrac{2x}{1+x^2}$ $\quad f'(x) = \dfrac{2(1+x^2) - 2x(2x)}{(1+x^2)^2} = \dfrac{-2x^2+2}{(1+x^2)^2}$

$$f''(x) = \frac{-4x(1+x^2)^2 - (-2x^2+2)(2(1+x^2)(2x))}{(1+x^2)^4}$$

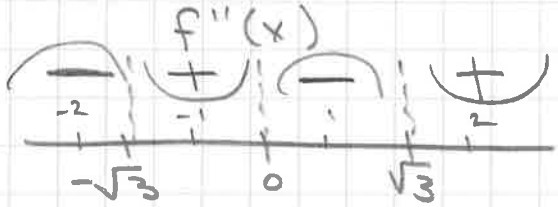
$$= \frac{(1+x^2)(-4x(1+x^2) - (-2x^2+2)(4x)}{(1+x^2)^4}$$

$$= \frac{4x^3 - 12x}{(1+x^2)^3}$$

critical #'s $\quad 4x^3 - 12x = 0$

$4x(x^2-3) = 0$

$x = 0 \quad x = \pm\sqrt{3}$



$\therefore$ $f(x)$ is concave down on $(-\infty, -\sqrt{3})$ & $(0, \sqrt{3})$

$f(x)$ is concave up $(-\sqrt{3}, 0)$ & $(\sqrt{3}, \infty)$

and $f(x)$ has points of inflection at $(-\sqrt{3}, f(-\sqrt{3}))$, $(0, f(0))$, & $(\sqrt{3}, f(\sqrt{3}))$

6. $f(x) = x^4 - 8x \quad [0,2]$

MVT: $f'(c) = \dfrac{f(b)-f(a)}{b-a}$ for $f(x)$ on $[a,b]$

$f'(x) = 4x^3 - 8 \qquad f(0) = 0 \qquad f(2) = 16 - 16 = 0$

Find where $f'(x) = \dfrac{0-0}{2-0} = \dfrac{0}{2} = 0$

$4x^3 - 8 = 0 \longrightarrow x = \sqrt[3]{2}$

when $x = \sqrt[3]{2}$ the avg slope over $[0,2]$ is equal to the slope at $x = \sqrt[3]{2}$ by the MVT because $f(x)$ is differentiable on $[0,2]$ & continuous on $[0,2]$

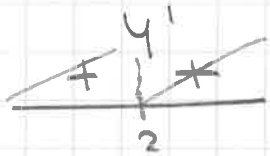
7. $y = \dfrac{5-3x}{x-2}$

D: all $\mathbb{R}$ except $x=2$
Vertical Asy. at $x=2$
Horizont Asy. at $y=-3$

$$y' = \frac{-3(x-2)-(5-3x)(1)}{(x-2)^2} = \frac{1}{(x-2)^2} = (x-2)^{-2}$$

$$y'' = -2(x-2)^{-3} = \frac{-2}{(x-2)^3}$$

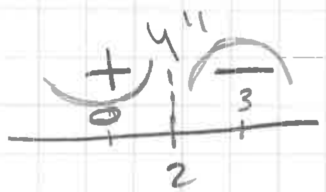
Critical #'s for y': $x-2=0$ $x=2$



y is increasing $(-\infty,2)$ & $(2,\infty)$

y has no relative extrema

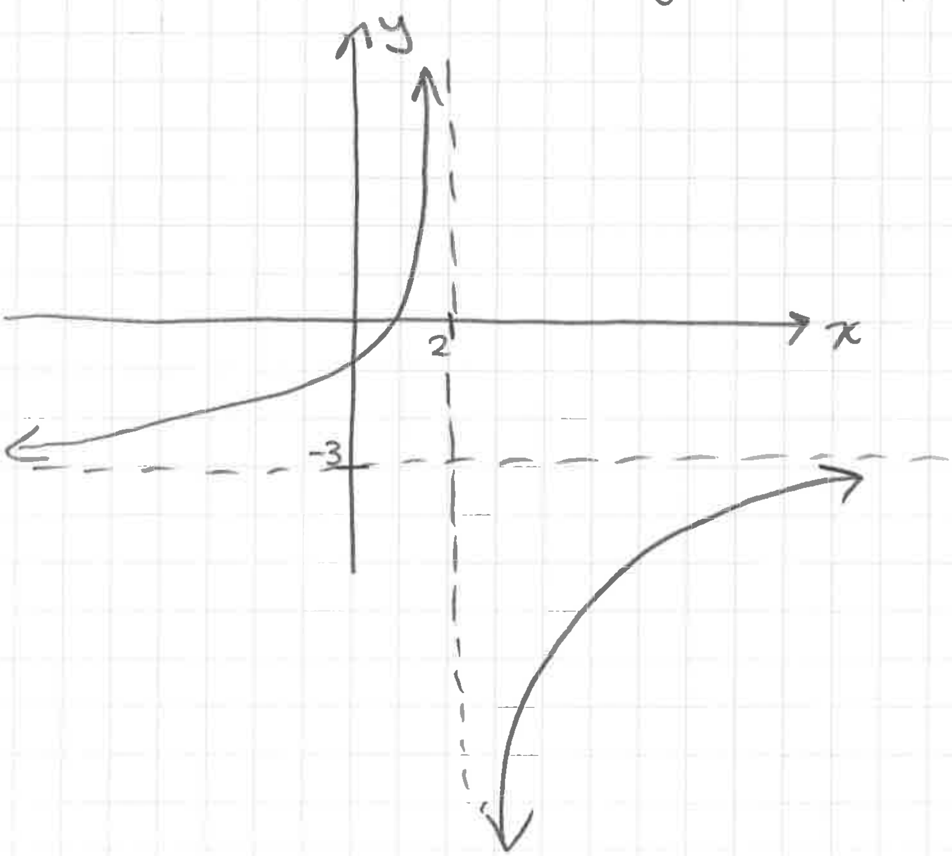
Critical #'s for y'': $x-2=0$ $x=2$



y is concave up $(-\infty,2)$

y is concave down $(2,\infty)$

y has no point of inflection



8. a $\lim_{x\to\infty} \frac{5-3x}{x-2} = \frac{-3}{1} = -3$ deg of numerator & denominator are the same.

b. $\lim_{x\to-\infty} \frac{3x^2+1}{1-x} = \infty$ deg of numerator is bigger

c. $\lim_{x\to-\infty} \frac{-3x}{\sqrt{x^2+4}}$

$= \lim_{x\to-\infty} \frac{\frac{-3x}{x}}{\frac{\sqrt{x^2+4}}{x}} = \lim_{x\to-\infty} \frac{-3}{\frac{\sqrt{x^2+4}}{-\sqrt{x^2}}}$

$= \lim_{x\to-\infty} \frac{-3}{-\sqrt{1+\frac{4}{x^2}}} = \frac{-3}{-\sqrt{1}} = 3$

9. Primary Equation → Surface Area of can → $A = 2\pi r^2 + 2\pi r h$

Secondary equation → Volume of can → $1000 = \pi r^2 h$

then $h = \frac{1000}{\pi r^2}$ & $A = 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$

$A = 2\pi r^2 + 2000 r^{-1}$

$A' = 4\pi r - 2000 r^{-2}$

$= \frac{4\pi r^3 - 2000}{r^2}$

Critical #'s:

$4\pi r^3 - 2000 = 0$ or $r^2 = 0$

$r = \sqrt[3]{\frac{500}{\pi}}$ $r = 0$

we can eliminate $r=0$

$A'' = 4\pi + 4000 r^{-3}$

since A'' is pos at $r = \sqrt[3]{\frac{500}{\pi}}$

By 2nd derivative Test $r = \sqrt[3]{\frac{500}{\pi}}$ is a min

& the dimensions required are $r = \sqrt[3]{\frac{500}{\pi}}$ & $h = 2\sqrt[3]{\frac{500}{\pi}}$