Calc I Test #1 Problem Set G. Buthusiem

1. Find the following limits, if they exist. If they do not exists, state whether the limit approachs $-\infty$, ∞, or DNE.

a. $\displaystyle\lim_{t\to 4} \frac{t-4}{t^2-16}$ b. $\displaystyle\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x}$ c. $\displaystyle\lim_{s\to 0} \frac{\frac{1}{\sqrt{1+s}}-1}{s}$

d. $\displaystyle\lim_{\theta\to 0} \frac{\cos\theta\tan\theta}{\theta}$ e. $\displaystyle\lim_{x\to 4^-} \frac{3x+4}{x^2-16}$

2. Find the derivative using the limit definition for $f(x)=x^3-12x$

3. Find the derivative of the following

a. $f(x)=\sqrt{x}\sin x$ b. $y=\dfrac{2x+7}{\sqrt{x^2+4}}$ c. $y=2x-x^2\tan x$

d. $y=\cot(5x^3+x)$ e. $y=(x^2-6)^3$ f. $y=\left(\dfrac{x+5}{x^2+3}\right)^2$

4. Find the equation of the line tangent to $y=(2x+3)^3$ at $x=1$.

5. Find $f^{(51)}(x)$ for $y=\cos x$

6. Find $\dfrac{dy}{dx}$ for $\sqrt{xy}=x-4y$

7. All the edges of a cube are expanding at a rate of 8 centimeters per second. How fast is the surface area changing when each edge is 6.5 centimeters?

8. If $s(t)=-16t^2+10t-8$ is a position function for a falling object find the velocity & acceleration of that object when it stops.

Calc I Test #1 Problem Set Answer KEY G. Bethusiem

1.a. $\lim_{t\to 4} \frac{t-4}{t^2-16} = \lim_{t\to 4} \frac{t-4}{(t+4)(t-4)} = \lim_{t\to 4} \frac{1}{t+4} = \frac{1}{4+4} = \frac{1}{8}$

2 b. $\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x} = \lim_{x\to 0} \frac{\sqrt{4+x}-2}{x} \cdot \frac{\sqrt{4+x}+2}{\sqrt{4+x}+2} = \lim_{x\to 0} \frac{4+x-4}{x(\sqrt{4+x}+2)}$

$= \lim_{x\to 0} \frac{1}{\sqrt{4+x}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{4}$

c. $\lim_{s\to 0} \frac{\frac{1}{\sqrt{1+s}} - 1}{s} = \lim_{s\to 0} \frac{\frac{1}{\sqrt{1+s}} - \frac{\sqrt{1+s}}{\sqrt{1+s}}}{s} = \lim_{s\to 0} \frac{1-\sqrt{1+s}}{s\sqrt{1+s}} =$

$\lim_{s\to 0} \frac{1-\sqrt{1+s}}{s\sqrt{1+s}} \cdot \frac{1+\sqrt{1+s}}{1+\sqrt{1+s}} = \lim_{s\to 0} \frac{1-(1+s)}{s\sqrt{1+s}(1+\sqrt{1+s})} = \lim_{s\to 0} \frac{-1}{\sqrt{1+s}(1+\sqrt{1+s})}$

$= \frac{-1}{\sqrt{1}(1+\sqrt{1})} = -\frac{1}{2}$

d. $\lim_{\theta\to 0} \frac{\cos\theta \tan\theta}{\theta} = \lim_{\theta\to 0} \frac{\cos\theta \cdot \frac{\sin\theta}{\cos\theta}}{\theta} = \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$

e. $\lim_{x\to 4^-} \frac{3x+4}{x^2-16} = -\infty$ 4 is a Vert. Asy. when approached from the left the function is negative

2. $f'(x) = \lim_{\Delta x\to 0} \frac{(x+\Delta x)^3 - 12(x+\Delta x) - (x^3-12x)}{\Delta x}$

$= \lim_{\Delta x\to 0} \frac{x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - 12x - 12\Delta x - x^3 + 12x}{\Delta x}$

$= \lim_{\Delta x\to 0} \frac{3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - 12\Delta x}{\Delta x}$

$= \lim_{\Delta x\to 0} 3x^2 + 3x\Delta x + \Delta x^2 - 12$

$= 3x^2 - 12$

3.a $f'(x) = \frac{d}{dx}[x^{1/2}]\sin x + x^{1/2}\frac{d}{dx}[\sin x] = \frac{1}{2}x^{-1/2}\sin x + x^{1/2}\cos x$

$$= \frac{\sin x}{2x^{1/2}} + x^{1/2}\cos x = \frac{\sin x}{2x^{1/2}} + \frac{2x^{1/2}}{2x^{1/2}}x^{1/2}\cos x$$

$$= \frac{\sin x + 2x\cos x}{2x^{1/2}}$$

b. $y' = \dfrac{\frac{d}{dx}[2x+7](x^2+4)^{1/2} - (2x+7)\frac{d}{dx}[(x^2+4)^{1/2}]}{(x^2+4)}$

$$= \frac{2(x^2+4)^{1/2} - (2x+7)\frac{1}{2}(x^2+4)^{-1/2}(2x)}{x^2+4}$$

$$= \frac{2(x^2+4)^{1/2} - \frac{2x^2+7x}{(x^2+4)^{1/2}}}{x^2+4} = \frac{2(x^2+4) - 2x^2 - 7x}{(x^2+4)^{3/2}}$$

$$= \frac{-7x+8}{(x^2+4)^{3/2}}$$

c. $\frac{dy}{dx} = 2 - \left(\frac{d}{dx}[x^2]\tan x + \frac{d}{dx}[\tan x]x^2\right) = 2 - 2x\tan x - x^2\sec^2 x$

d. $y' = -\csc^2(5x^3+x)\cdot\frac{d}{dx}[5x^3+x]$

$$= -(15x^2+1)\csc^2(5x^3+x)$$

e. $y' = 3(x^2-6)^2\cdot\frac{d}{dx}[x^2-6] = 6x(x^2-6)^2$

f. $\frac{dy}{dx} = 2\left(\frac{x+5}{x^2+3}\right)\cdot\frac{d}{dx}\left[\frac{x+5}{x^2+3}\right] = \frac{2(x+5)}{x^2+3}\cdot\left(\frac{1(x^2+3)-(x+5)(2x)}{(x^2+3)^2}\right)$

$$= \frac{2(x+5)(-x^2-10x+3)}{(x^2+3)^3}$$

4. $m_{tan} = y' = 3(2x+3)^2(2)$

m_{tan} at $x=1$ $= y'(1) = 3(2(1)+3)^2(2) = 150$

if $x=1$ $y(1) = (2(1)+3)^3 = 125$ so

equation of tan line

$y - y_1 = m(x - x_1) \rightarrow y - 125 = 150(x-1) \rightarrow \boxed{y = 150x - 25}$

5. $f(x) = \cos x$ Remainder 0

$f'(x) = -\sin x$ 1

$f''(x) = -\cos x$ 2

$f'''(x) = \sin x$ 3

$f^{(4)}(x) = \cos x$ 0

$51 = 4 \times 12 + 3$

$\therefore f^{(51)}(x) = \sin x$

6. $(xy)^{1/2} = x - 4y$

$$\frac{d}{dx}\left[(xy)^{1/2}\right] = \frac{d}{dx}\left[x - 4y\right]$$

$$\frac{1}{2}(xy)^{-1/2} \cdot \frac{d}{dx}[xy] = 1 - 4\frac{dy}{dx}$$

$$\frac{1}{2(xy)^{1/2}} \cdot \left(y + x\frac{dy}{dx}\right) = 1 - 4\frac{dy}{dx}$$

$$\frac{y}{2(xy)^{1/2}} + \frac{x}{2(xy)^{1/2}}\frac{dy}{dx} = 1 - 4\frac{dy}{dx}$$

$$\left(4 + \frac{x}{2(xy)^{1/2}}\right)\frac{dy}{dx} = 1 - \frac{y}{2(xy)^{1/2}}$$

$$\frac{dy}{dx} = \frac{\frac{2(xy)^{1/2} - y}{2(xy)^{1/2}}}{\frac{8(xy)^{1/2} + x}{2(xy)^{1/2}}}$$

$$\frac{dy}{dx} = \frac{2(xy)^{1/2} - y}{8(xy)^{1/2} + x}$$

7. ~~Volume of cube~~

$$\cancel{V = s^3}$$

$$\cancel{\frac{dV}{dt} = 3s^2\frac{ds}{dt}}$$

Surface area of cube

$$SA = 6s^2$$

$$\frac{dSA}{dt} = 12s\frac{ds}{dt}$$

$$\frac{dSA}{dt} = 12(6.5)\cdot(8) =$$

8. $s(t) = -16t^2 + 10t - 8$

$v(t) = s'(t) = -32t + 10$

$a(t) = s''(t) = -32$

When it stops $v(t) = 0$
so the velocity is 0

but $v(t) = 0$ when
$-32t + 10 = 0$

$t = \frac{5}{16}$ which is
the point in time when it stops

so
$a(5/16) = -32$

and the acceleration when it stops is -32